

Implications of Market Power for Financial Frictions *

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Abstract

Should policies eliminating markups account for financial frictions? I study this in a macro-finance model with variable markups and earnings-based borrowing constraints. The two distortions interact: eliminating markups depresses profits, tightens borrowing constraints, and increases capital misallocation. This interaction accounts for 25 percent of total misallocation, a loss of 2.1 percent in aggregate productivity, despite financial frictions being modest in isolation. A planner constrained by financial frictions optimally allows markup dispersion, trading greater markup dispersion for reduced dispersion in the marginal product of capital. Each firm's socially optimal markup approaches the competitive equilibrium level as its cost of financing increases.

Keywords: market power, financial frictions, misallocation, optimal policy

JEL classifications: D4, E1, G1

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1 Introduction

Markups are a major source of inefficiency. In the United States, [De Loecker et al. \(2020\)](#) estimate the aggregate markup to be approximately 1.40 in recent years, and dispersion in firm-level markups generates substantial misallocation ([Edmond et al. 2023](#)). A natural policy prescription is to eliminate markups. Yet firms also face financial frictions, and markups generate profits that help firms self-finance investment and relax borrowing constraints. When both distortions are present, eliminating markups may tighten financial constraints and increase capital misallocation. The gains from reduced market power can be partly offset by greater distortions in the allocation of capital. This paper asks whether policies that eliminate markups remain optimal in the presence of financial frictions.

I find that eliminating markups is suboptimal. In a quantitative model calibrated to U.S. non-financial corporations, total productivity losses from misallocation are 8.3 percent. Of these, approximately 25 percent is attributable to the interaction of markups and financial frictions, a loss of 2.1 percent in aggregate productivity, even though financial frictions generate only a 1 percent loss in isolation. The interaction arises because eliminating markups reduces profits significantly, tightening earnings-based borrowing constraints and leaving more firms financially constrained for longer. Hence, the productivity gains from eliminating markups are counteracted by capital misallocation.

Optimal intervention in product markets would not eliminate all markups. Using a flexible subsidy schedule that nests the markup-eliminating policy as a special case, I solve for the welfare-maximizing policy along the transition path. The optimal policy trades greater dispersion in markups for reduced dispersion in the marginal product of capital, delivering welfare gains roughly 1 percentage point larger in consumption-equivalent terms than a policy that eliminates markups entirely. Approximately 25 percent of the markup dispersion in the competitive equilibrium is retained under the optimal policy.

I confirm and generalize these findings by characterizing the constrained-efficient allocation. A planner subject to financial frictions sets each firm's markup such that firms facing severe financing costs are allowed markups closer to the competitive equilibrium level, because the resulting profits relax their borrowing constraints. This leads to a reduction in capital misallocation at the cost of some dispersion in markups.

Model Overview. I study a macro-finance model in the tradition of [Khan and Thomas \(2013\)](#). Firms are monopolistically competitive. They sell goods at a markup generated by a [Kimball \(1995\)](#) aggregator. Firms are subject to two types of financial frictions. First, borrowing is limited by an earnings-based borrowing constraint. The earnings-based constraint

caps the ratio of debt to current earnings, following evidence in [Lian and Ma \(2021\)](#) that this is the dominant form of corporate borrowing in the United States. Second, dividends are restricted to being non-negative. I calibrate the model to match key features of U.S. non-financial corporations between 2001 and 2019, including leverage, sales concentration, and the aggregate markup.

Decomposition. To quantify the interaction, I implement two policies that target each distortion independently: credit subsidies to eliminate financial frictions and size-dependent subsidies to eliminate the level and dispersion of markups. Implemented together, the policies recover the first-best allocation. Implemented separately, they decompose total productivity losses into components due to each distortion alone and a residual reflecting their interaction. Eliminating markups alone raises productivity by 5.3 percent but cannot close the remaining gap to the frontier. The interaction is asymmetric: eliminating markups substantially worsens financial frictions, as lower profit rates extend the time firms remain constrained and increase the dispersion of the marginal product of capital. On the other hand, eliminating financial frictions has little effect on product market distortions.

Optimal Policy. To disentangle how markups interact with financial frictions, I generalize the size-dependent subsidy to allow independent control over the level and dispersion of markups. The dispersion of markups, rather than the aggregate markup level, is responsible for alleviating financial frictions. Varying the aggregate markup while holding dispersion fixed has negligible effects on capital misallocation but reduces welfare through depressed wages. The optimal policy within this subsidy schedule retains significant markup dispersion, roughly 25 percent of that in the competitive equilibrium, while reducing the aggregate markup below one.

To confirm these findings beyond the particular subsidy schedule, I characterize the constrained-efficient allocation. A Ramsey planner maximizes the lifetime utility of the representative household, choosing firm-level capital, debt, and output decisions, subject to the same earnings-based borrowing constraints and non-negative dividend constraints that firms face in the competitive equilibrium. The question is whether a planner facing these constraints would eliminate markups or instead allow some firms to price above marginal cost. I find that in the constrained-efficient allocation, markups are a harmonic average of the markups of an unconstrained firm and the competitive equilibrium markups, the latter expressed as a Lerner index. This harmonic average is weighted by the firm-specific shadow cost of the financial frictions. The planner allows prices above marginal cost for highly constrained firms because the resulting profits relax their borrowing constraints and reduce

capital misallocation.

The constrained-efficient allocation can be decentralized through firm-specific transfers. These transfers resemble the size-dependent subsidies that eliminate markups. However, the subsidy a firm receives is scaled down in proportion to the shadow cost of its financial frictions. Intuitively, transferring resources to constrained firms to eliminate their markups is counterproductive. Doing so removes the profits these firms need to relax their borrowing constraints. Policies that eliminate markups therefore transfer too much to precisely the firms for which markups are most beneficial.

Related Literature. I contribute to three strands of the literature. The first is the literature on financial frictions and misallocation. I include a financial block as in [Khan and Thomas \(2013\)](#). Building on the misallocation framework of [Restuccia and Rogerson \(2008\)](#) and [Hsieh and Klenow \(2009\)](#), previous work including [Buera and Shin \(2013\)](#), [Midrigan and Xu \(2014\)](#), and [Gopinath et al. \(2017\)](#) has studied how financial frictions generate productivity losses through capital misallocation. I introduce market power into this setting and study its interaction with financial frictions and its implications for aggregate productivity.

Recent work by [Li et al. \(2025\)](#) independently studies this interaction, documenting that less constrained firms charge higher markups and that this relationship is stronger in industries where firms rely more on earnings-based borrowing. They emphasize two-way feedback between markups and borrowing constraints that contributes to misallocation. Relative to their work, I characterize the constrained-efficient allocation and solve for optimal product market policy. The choice of an earnings-based borrowing constraint follows evidence in [Lian and Ma \(2021\)](#) that the majority of U.S. corporate borrowing is earnings-based. [Drechsel \(2023\)](#) shows that earnings-based constraints are important for macroeconomic fluctuations. This specification is central to my results, because the borrowing limit depends on current earnings; markups therefore expand borrowing capacity through their effect on profits.¹

The second is the literature on markups and misallocation. [Edmond et al. \(2023\)](#) characterize the costs of variable markups in terms of entry, misallocation, and the aggregate markup. [Baqae and Farhi \(2020\)](#) develop a general framework for measuring the aggregate costs of markup dispersion. These analyses abstract from financial frictions. I study how their conclusions change when financial frictions are present. [Boar and Midrigan \(2023\)](#) incorporate both markups and financial frictions in a model of heterogeneous entrepreneurs to study the consequences of product market interventions for inequality. I focus instead

¹Markups also relax the non-negative dividend constraint by increasing cash flow, allowing firms to self-finance investment. Hence, the interaction can also be present under asset-based borrowing constraints.

on productivity and characterize the constrained-efficient allocation and optimal product market policy.

The third is the literature on policy design under financial frictions. [Guner et al. \(2008\)](#) find that size-dependent policies limiting firm size carry significant output costs. [Buera and Shin \(2017\)](#) show that financial frictions limit the gains from reforms that eliminate size-dependent distortions. I show that financial frictions similarly constrain size-dependent subsidies aimed at curbing market power. [Dávila and Korinek \(2018\)](#) characterize pecuniary externalities and constrained-efficient policy in economies with financial frictions. [Lanteri and Rampini \(2023\)](#) and [Itskhoki and Moll \(2019\)](#) study constrained-efficient capital reallocation in such environments. These papers abstract from market power. I show that when variable markups are present, the constrained-efficient policy becomes size-dependent: transfers to firms vary with the severity of their financial frictions.

2 Model

The model extends [Khan and Thomas \(2013\)](#) by introducing variable markups and an earnings-based borrowing constraint. Time is discrete and there is no aggregate uncertainty. A representative household consumes a final good, supplies labor, and owns all firms. The final good is produced by aggregating intermediate goods through a [Kimball \(1995\)](#) aggregator, which generates variable markups. Intermediate goods are produced by monopolistically competitive firms that make capital investment, borrowing, and output decisions to maximize the present discounted value of dividends. Credit supply is limited by an earnings-based constraint.

2.1 Representative Household

The representative household is infinitely lived, with period utility $U(C, L) = \log C - \Psi L$ over consumption of the final good C and labor supply L . The household discounts the future at rate $\beta \in (0, 1)$, borrows and saves at price q_b , and owns all firms, receiving dividends $d(z, k, b)$ from each firm in state (z, k, b) . The household solves the following dynamic problem:

$$\begin{aligned} V(a) &= \max U(C, L) + \beta V(a') \\ \text{s.t.} \quad & C + q_b a' \leq a + WL + \int d(z, k, b) d\mu(z, k, b) \end{aligned}$$

From the first-order conditions: $q_b = \beta \frac{C}{C'}$ and $W = \Psi C$.

2.2 Final Good

A continuum of firms, indexed by $i \in [0, 1]$, produce intermediate goods y_i that are aggregated into a final good Y implicitly defined by

$$\int \Upsilon \left(\frac{y_i}{Y} \right) di = 1$$

where $\Upsilon'(q) > 0$ and $\Upsilon''(q) < 0$ as in [Kimball \(1995\)](#). Let relative output be $q \equiv \frac{y}{Y}$. Cost minimization yields the demand curve

$$p(q_i) = \Upsilon'(q_i)D$$

where $p(q)$ is the price of the intermediate good and D is the demand shifter, defined by

$$D = \left[\int q_i \Upsilon'(q_i) di \right]^{-1}$$

Intermediate good firms are monopolistic competitors and take their contribution to D as given when optimizing. The elasticity of demand and markups are, respectively,

$$\theta(q) \equiv -\frac{\partial \log q}{\partial \log p} = \frac{-\Upsilon'(q)}{\Upsilon''(q)q} \quad m(q) \equiv \frac{\theta(q)}{\theta(q) - 1}$$

I adopt the [Klenow and Willis \(2016\)](#) parameterization of the Kimball aggregator, which is summarized by two parameters:² σ , governing the elasticity of demand at $q = 1$, and ε , governing how the elasticity varies with firm size. Under this parameterization,

$$\theta(q) = \sigma q^{-\frac{\varepsilon}{\sigma}} \quad m(q) = \frac{\sigma}{\sigma - q^{\frac{\varepsilon}{\sigma}}}$$

The super-elasticity of demand is $\frac{\partial \log \theta(q)}{\partial \log q} = -\frac{\varepsilon}{\sigma} < 0$: as firms increase their relative size, demand becomes less elastic and markups rise. If $\varepsilon = 0$, the model reduces to constant elasticity of substitution (CES) with $\theta(q) = \sigma$ and $m(q) = \frac{\sigma}{\sigma-1}$ for all q .

²Where

$$\Upsilon(q) = 1 + (\sigma - 1) \exp \left(\frac{1}{\varepsilon} \right) \varepsilon^{\frac{\sigma}{\varepsilon} - 1} \left[\Gamma \left(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon} \right) - \Gamma \left(\frac{\sigma}{\varepsilon}, \frac{q^{\varepsilon/\sigma}}{\varepsilon} \right) \right]$$

and $\Gamma(y, x) = \int_x^\infty t^{y-1} \exp(-t) dt$.

2.3 Intermediate Good

Intermediate good firms are monopolistically competitive. Each firm has idiosyncratic productivity z and produces output using a Cobb-Douglas technology with constant returns to scale,

$$y = zk^\alpha \ell^{1-\alpha}$$

that combines labor ℓ and capital k with capital share $\alpha \in (0, 1)$. Productivity is a persistent state and evolves according to

$$z' = \begin{cases} \sim F(z') & \text{with prob. } 1 - \rho_z \\ z & \text{with prob. } \rho_z \end{cases} \quad (1)$$

where $F(z)$ is the Pareto distribution with shape parameter ζ . The variable ρ_z characterizes the persistence of productivity.

Static decisions. Firms choose output taking capital as given. Profits from production for a firm with capital k and productivity z are

$$\pi(z, k) = \max_y p(y)y - W\ell$$

The first-order condition in terms of relative output solves:

$$p(q) = m(q) \frac{1}{q} \frac{WY^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \left(\frac{q}{zk^\alpha} \right)^{\frac{1}{1-\alpha}}$$

The optimal price is a markup over the marginal cost, where $m(q)$ is as defined above. Define $q(z, k)$ as the relative output policy of a firm with productivity z and capital k .

Dynamic decisions. Firms are characterized by their state (z, k, b) , where b is the outstanding debt balance. Each period, firms issue dividends d to the representative household, choose next-period capital k' and debt b' , and pay fixed operating costs $f_c > 0$. Capital is chosen before the realization of next-period productivity and depreciates at rate $\delta \in (0, 1)$. Firms borrow at price q_b from the representative household. The budget constraint is

$$d \leq \pi(z, k) - b + (1 - \delta)k - f_c - k' + q_b b'$$

Firms face two financial constraints. First, they are subject to an earnings-based borrowing constraint (EBC), where the maximum ratio of debt to earnings is $\phi > 0$:

$$b' \leq \phi \pi(z, k)$$

I focus on earnings-based constraints as the benchmark financial friction, following evidence in [Lian and Ma \(2021\)](#) that the majority of U.S. corporate borrowing takes this form. Second, dividends must be non-negative, $d \geq 0$. This imposes an equity issuance constraint: firms cannot raise external equity and must finance investment from retained earnings or debt.³

Firms exit exogenously with probability $\pi_d \in (0, 1)$ and learn of exit before the period begins. In their final period, exiting firms produce, liquidate their capital stock, and repay outstanding debt. Exiting firms are replaced by entrants with state (z_0, k_0, b_0) where $z_0 \sim F_0(z)$. The firm's problem can be stated recursively as

$$\begin{aligned} V(z, k, b) &= \max_{d, k', b'} d + \beta(1 - \pi_d) \mathbb{E}[V(z', k', b') | z] + \beta \pi_d \mathbb{E}[\pi(z', k') + (1 - \delta)k' - b' - f_c | z] \quad (2) \\ \text{s.t.} \quad & d \leq \pi(z, k) - b + (1 - \delta)k - f_c - k' + q_b b' \\ & d \geq 0 \\ & b' \leq \phi \pi(z, k) \end{aligned}$$

Let $\mu(z, k, b)$ be the distribution of firms actively producing across states (z, k, b) . It is consistent with the law of motion described in detail in [Appendix A.1](#). The stationary equilibrium is defined in [Appendix A.2](#).

2.4 Firms' Policies

If firms are unconstrained by the dividend and borrowing constraints, they are indifferent over debt choice. Following [Khan and Thomas \(2013\)](#), I fix the debt policy of unconstrained firms such that they follow a maximum borrowing policy. This means that unconstrained firms choose the maximum level of debt consistent with remaining unconstrained in all future states.

Define $k^*(z)$ as the capital choice of unconstrained firms. The capital choice of unconstrained firms solves the following:

$$\frac{1}{\beta} - (1 - \delta) = \mathbb{E} \left[\frac{\partial \pi(z', k^*(z))}{\partial k} \middle| z \right] \quad (3)$$

³This assumption, combined with the borrowing constraint, ensures that financial structure is not irrelevant as in Modigliani-Miller.

This level of capital equates the user cost of capital, $\frac{1}{\beta} - (1 - \delta)$, with the expected marginal product of capital. Let $b^*(z)$ be the minimum savings plan. To ensure dividends are positive next period, $b^*(z)$ solves the following

$$b^*(z) = \min_{z'} \pi(z', k^*(z)) + (1 - \delta)k^*(z) - f_c + q_b \min\{b^*(z'), \phi\pi(z', k^*(z))\} - k^*(z') \quad (4)$$

Define the net worth of a firm as $n(z, k, b) = \pi(z, k) - b + (1 - \delta)k - f_c$. The following proposition summarizes the debt and capital policy of firms.

Proposition 1. *For a firm in state (z, k, b)*

- *If $n(z, k, b) \geq k^*(z) - q_b b^*(z)$, then the firm is **unconstrained**, $k'(z, k, b) = k^*(z)$, $b'(z, k, b) = b^*(z)$, and $d = n(z, k, b) + q_b b^*(z) - k^*(z)$*
- *If $n(z, k, b) < k^*(z) - q_b b^*(z)$, then the firm is **constrained** and the dividend constraint binds, $d = 0$.*

Details on the proposition can be found in Appendix A.3.

2.5 Aggregation

I define two aggregate wedges that summarize the distortions from markups and financial frictions, respectively. The aggregate markup $\mathcal{M}$ is implicitly defined by

$$\frac{WL}{Y} = \frac{1 - \alpha}{\mathcal{M}} \quad (5)$$

It can be expressed as the sales-weighted harmonic mean of firm-level markups

$$\mathcal{M} = \left[\int \frac{p(q(z, k))q(z, k)}{m(q(z, k))} d\mu(z, k) \right]^{-1}$$

To capture distortions in the allocation of capital, I define the capital wedge $\mathcal{U}$ analogously through the capital share of income

$$\frac{RK}{Y} = \frac{\alpha}{\mathcal{U}} \quad (6)$$

where $R = \frac{1}{\beta} - (1 - \delta)$ is the user-cost of capital. The capital wedge reflects both markups and financial frictions. To express it, define $\Delta_k(z, k, b)$ as a firm-specific capital wedge solving

$$\frac{1}{\beta} - (1 - \delta) = \mathbb{E} \left[\frac{\partial \pi(z', k'(z, k, b))}{\partial k} \Delta_k(z, k, b) \middle| z \right]$$

where $k'(z, k, b)$ is the capital policy for a firm. If the firm is unconstrained then $\Delta_k(z, k, b) = 1$. Otherwise, smaller values of $\Delta_k(z, k, b)$ imply the firm faces greater financial frictions. Dispersion in $\Delta_k(z, k, b)$ maps directly to dispersion in expected marginal products of capital. The aggregate capital wedge can be expressed as a harmonic average of $\Delta_k(z, k, b)$ and markups:⁴

$$\mathcal{U} \approx \left[\int p(q(z, k)) q(z, k) \frac{\Delta_k(z, k, b)}{m(q(z, k))} d\mu(z, k, b) \right]^{-1}$$

When financial frictions are absent, $\Delta_k(z, k, b) = 1$ for all firms and $\mathcal{U} \approx \mathcal{M}$. Therefore, one indication of the contribution of financial frictions to misallocation will be the relative size of $\mathcal{U}$ to $\mathcal{M}$. As it increases, financial frictions distort the allocation of capital across firms.

The economy aggregates to $Y = ZK^\alpha L^{1-\alpha}$ where Z is aggregate productivity. An expression for Z , along with all the aggregation results, is provided in Appendix A.4.

3 Quantitative Analysis

This section uses the calibrated model to study the interaction between financial frictions and markups. I decompose total misallocation into contributions from each distortion and their interaction, compare the effects of credit and size-dependent subsidies in detail, and solve for the optimal product market policy. The model is calibrated to match key features of the U.S. non-financial corporate sector between 2001 and 2019.

3.1 Calibration

Externally Calibrated. The model is calibrated at an annual frequency. Table 1 summarizes externally calibrated parameters. The discount factor is $\beta = 0.96$, the capital share is $\alpha = 0.33$, and the depreciation rate is $\delta = 0.06$. Labor disutility Ψ is chosen so that $L = 0.6$ in the baseline economy. The super-elasticity is $\varepsilon/\sigma = 0.50$, a conservative midpoint between the estimates implied by [Amiti et al. \(2019\)](#) and [Edmond et al. \(2023\)](#).⁵ Persistence of productivity follows [Buera et al. \(2011\)](#). The exogenous exit rate is $\pi_d = 0.10$ as in [Khan and Thomas \(2013\)](#).

⁴This is an approximation due to the very persistent productivity process in the calibration exercise to follow. Details found in Appendix A.4.

⁵In [Edmond et al. \(2023\)](#), $\varepsilon/\sigma = 0.16$. In [Amiti et al. \(2019\)](#), the benchmark pass-through for strategic complements is $\frac{\Gamma}{1+\Gamma} \approx 0.5$, where for the [Klenow and Willis \(2016\)](#) version of [Kimball \(1995\)](#) preferences $\Gamma = \frac{\varepsilon}{\sigma-1}$. For $\sigma \approx 7.0$, which delivers the aggregate markup target in all calibrations, their results imply $\varepsilon/\sigma \approx 0.86$.

Table 1: Externally Calibrated Parameters

Parameter	Value	Description	Source
β	0.960	Discount Factor	-
Ψ	1.064	Labour Disutility	-
α	0.330	Capital Share	-
δ	0.060	Depreciation Rate	-
ε/σ	0.500	Super Elasticity	Amiti et al. (2019)
ρ	0.89	TFP Persistence	Buera et al. (2011)
π_d	0.100	Exit Rate	Khan and Thomas (2013)
k_0	1.385	Entrant Capital	-
b_0	0.000	Entrant Debt	-
$\bar{z}_0$	1.119	Entrant Max TFP	-

For entry, I truncate the initial productivity distribution $F_0(z)$ at the 25th percentile of the Pareto distribution $F(z)$, generating life-cycle dynamics in which entrants' productivity grows over time. This implies that entrants account for 3 percent of employment, consistent with the Census Bureau's Business Dynamics Statistics. Entrants begin with capital $k_0 = 1.385$, which is 25 percent below the smallest unconstrained capital choice, ensuring that entrants are financially constrained. This implies entrants' capital share is 6 percent. Entrants have no outstanding debt, $b_0 = 0$. These choices keep the entry margin in line with the literature.⁶

Table 2: Internally Calibrated Parameters

Parameter	Value	Description
ϕ	3.982	Debt/Earnings Constraint
σ	8.881	Elasticity of Demand at $q = 1$
f_c	0.120	Fixed Cost
ζ	2.564	Pareto Shape

⁶See [Khan and Thomas \(2013\)](#), [Jo and Senga \(2019\)](#), and [Ottonello and Winberry \(2020\)](#), where entrants have no outstanding debt and their capital share ranges between 5-10 percent.

Internally Calibrated. Four parameters are calibrated to match moments in the data: ϕ , the debt-to-earnings ratio; σ , the elasticity of demand at $q = 1$; f_c , the fixed operating cost; and ζ , the Pareto shape parameter. I choose these to minimize the distance to four target moments: average leverage, the aggregate markup, the fraction of firms with a positive debt balance, and the sales share of the top 5 percent of firms.

Average leverage is the mean of b/k in Compustat for non-financial firms from 2001:Q1 to 2019:Q4.⁷ The target fraction of firms with positive debt is 84 percent.⁸ The aggregate markup target is $\mathcal{M} = 1.30$, within the range of plausible estimates in the literature.⁹ The sales concentration target is the median sector’s sales share of the top 5 percent of firms in the same Compustat panel. Table 2 reports the internally calibrated parameter values. Model moments and their targets are presented in Table 3a.

Table 3: Calibration Moments

(a) Targeted Moments

Moment	Model	Data	Source
$\mathbb{E}[\frac{b}{k}]$	0.25	0.24	Compustat
$\mathcal{M}$	1.30	1.30	–
$\mathbb{E}[\mathbb{I}(b > 0)]$	0.83	0.84	Compustat
Top 5 % sales-share	0.37	0.35	Compustat

(b) Non-Targeted Moments

Moment	Model	Data	Source
K/Y	2.29	2.30	BEA
$\mathbb{E}[\Delta \log(k)]$	0.06	0.04	Compustat
$SD[\Delta \log(k)]$	0.30	0.20	Compustat

Validation. Table 3b compares non-targeted moments. The model captures the aggregate capital-to-output ratio and average investment rate well. The model overstates the standard deviation of investment. The calibrated debt-to-earnings constraint ϕ is close to the median

⁷See Appendix B for details.

⁸Very similar to the value of 81 percent in Crouzet and Mehrotra (2020).

⁹See De Loecker et al. (2020), Edmond et al. (2023), or Demirer (2022).

values for financial covenants reported in [Drechsel \(2023\)](#), providing additional validation.

3.2 Misallocation

I measure misallocation following [Hsieh and Klenow \(2009\)](#) and [Midrigan and Xu \(2014\)](#). The efficient allocation is that of a social planner with the same aggregate capital and labor who chooses firm-level capital and output to maximize aggregate output Y . The planner is subject to the time-to-build restriction on capital and the entry-exit primitives of the model: firms exit exogenously at rate π_d and are replaced by entrants with productivity $z \sim F_0$ and initial capital k_0 . Misallocation is the log-difference between aggregate productivity under the planner's allocation and in the competitive equilibrium. The planner's problem and its solution are detailed in [Appendix A.5](#). The key properties are: (1) output is allocated such that firms price at marginal cost, eliminating markups, and (2) capital is allocated in proportion to expected sales.

Policies. The model features two sources of distortion: financial frictions and variable markups. I introduce two policies to target each independently. Implemented together, they recover the efficient allocation. Implemented separately, they allow me to decompose total productivity losses.

(1) Size-Dependent Subsidies. To eliminate variable markups, I introduce a size-dependent subsidy as in [Edmond et al. \(2023\)](#). A firm with relative output q receives the transfer

$$T(q) = DY [\Upsilon(q) - \Upsilon'(q)q] \quad (7)$$

This subsidy replaces marginal revenue with marginal utility in the firm's output decision, so that prices equal marginal cost. The aggregate markup falls to one and markup dispersion is eliminated. The total cost of the subsidy is funded by a lump-sum tax on the household.

(2) Credit Subsidy. To overcome financial frictions, a credit subsidy is provided to firms such that the first-best capital is attained. A firm in state (z, k, b) receives subsidy $k^*(z) - k'(z, k, b)$ if it is constrained and zero otherwise. The total cost of the transfer is $T = \int \max \{k^*(z) - k'(z, k, b), 0\} d\mu(z, k, b)$, which is funded by a lump-sum transfer of size T from the representative household.¹⁰

¹⁰[Jo and Senga \(2019\)](#) study the aggregate implications of these credit policies.

Decomposing Productivity Losses. Define Z^f as aggregate productivity when firms receive the credit subsidy and Z^d as productivity when the size-dependent subsidy is in place. Total productivity losses decompose as

$$\begin{array}{ccccccc}
\underbrace{\log(Z) - \log(Z^p)}_{\text{Total Losses}} & = & \underbrace{\log(Z) - \log(Z^f)}_{\checkmark \text{ Markups, } \times \text{ Financial Frictions}} & + & \underbrace{\log(Z) - \log(Z^d)}_{\times \text{ Markups, } \checkmark \text{ Financial Frictions}} & + & \underbrace{\Delta}_{\text{Interaction}} \\
-8.34\% & & -1.04\% & & -5.29\% & & -2.10\% \\
& & & & & & (8)
\end{array}$$

The first term measures how much of misallocation losses are attributable to financial frictions alone. The second term measures the analogous losses from eliminating markups while financial frictions remain active. The residual Δ captures their interaction.¹¹

Total productivity losses are 8.3 percent. Eliminating financial frictions through credit subsidies closes only 1 percent of the gap, suggesting that financial frictions are modest in isolation. Eliminating markups raises productivity by 5.3 percent but cannot reach the frontier. The interaction accounts for 2.1 percent, approximately 25 percent of total losses and twice the magnitude of financial frictions alone.

Taking the model as representative of the U.S. corporate sector, financial frictions appear to play a small role in isolation. However, as I will emphasize next, the interaction term can be attributed to financial frictions. Ignoring the interaction understates the losses due to financial frictions by a factor of three.

Robustness. Results are robust to replacing the earnings-based constraint with an asset-based constraint, $b' \leq \phi k$. Under an identical calibration, total losses are 10.2 percent, losses due to financial frictions are 1.7 percent, losses due to markups are 6.6 percent, and the interaction is 1.9 percent.

3.3 Policies

I now examine each policy in detail to characterize the source of the interaction. Table 4 compares the baseline economy to the economy with a credit subsidy. Table 5 does the same for the size-dependent subsidy that eliminates markups.

¹¹Equation (8) is an approximation. Z^p is computed using K from the competitive equilibrium, leading to inaccuracy since K changes when policy is implemented. If $\pi_d = 0$, Z^p becomes scale-invariant and the decomposition is exact. Since $\pi_d = 0.10$, the inaccuracies are small.

Table 4: Credit Subsidy

	Baseline	Credit Subsidy
Y	1.977	2.087
K	4.524	5.195
L	0.600	0.598
I	0.271	0.286
C	1.586	1.680
W	1.687	1.788
Productivity and Welfare		
$\% \Delta Z$	—	1.040
CEV	—	0.062
T/Y	—	0.621
Cross-section		
% Constrained	90	-
Top 5% sales-share (%)	37	40
$\mathcal{M}$	1.31	1.31
$SD(\log(m))$	0.114	0.113

Credit Subsidy. Table 4 confirms that financial frictions are not a significant source of productivity losses in isolation. Aggregate output increases by 6 percent, driven by higher capital and labor input. Welfare increases by 6 percent in consumption-equivalent terms.¹² Given the modest gains, there is very little change in the concentration of sales, the aggregate markup, or markup dispersion. Overall, eliminating financial frictions does not meaningfully affect the product market.

Size-Dependent Subsidy. Table 5 tells a different story. Eliminating markups raises output and wages by 60 percent, a byproduct of reducing the aggregate markup to one. Welfare increases by 28 percent in consumption-equivalent terms. The subsidized economy is substantially more concentrated: the top 5 percent sales share rises from 37 to 54 percent.

However, eliminating markups does not bring the economy to the frontier. Capital allocation worsens: the fraction of constrained firms increases from 90 to 96 percent. To understand why, recall from Proposition 1 that a firm is financially constrained when its net worth falls below the threshold $k^*(z) - q_b b^*(z)$. Figure 1 illustrates how the size-dependent subsidy affects this threshold. Eliminating markups promotes greater output, which requires

¹²CEV measured as Δ such that $\log(C(1 + \Delta)) - \Psi L = \log(\tilde{C}) - \Psi \tilde{L}$ between two steady states.

Table 5: Size-Dependent Subsidy

	Baseline	Size-Dependent Subsidy
Y	1.977	3.153
K	4.524	8.944
L	0.600	0.795
I	0.271	0.537
C	1.586	2.496
W	1.687	2.656
Productivity and Welfare		
$\% \Delta Z$	—	5.286
CEV	—	0.279
T/Y	—	0.387
Cross-section		
% Constrained	90	96
Top 5% sales-share (%)	37	54
$\log(\mathcal{U}/\mathcal{M})$	0.084	0.107
$Skew\left(\log \frac{1}{\Delta_k}\right)$	0.614	0.994
$SD\left(\log \frac{1}{\Delta_k}\right)$	0.183	0.193
$SD_s\left(\log \frac{1}{\Delta_k}\right)$	0.150	0.239
$Corr\left(k, \log \frac{1}{\Delta_k}\right)$	-0.283	0.008

substantially more capital. The unconstrained capital policy $k^*(z)$ increases by an order of magnitude. At the same time, profit rates are lower because firms price at marginal cost. Firms therefore need more financing to reach their first-best capital but accumulate internal funds more slowly. Eliminating markups therefore leaves more firms constrained for longer.

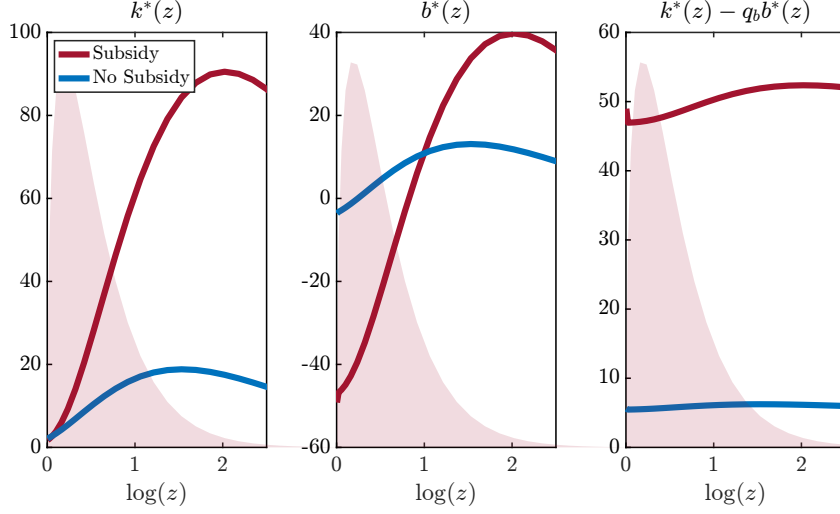
This is reflected in the distribution of capital wedges. Recall that capital wedges $\Delta_k(z, k, b)$ are defined as

$$\left[\frac{1}{\beta} - (1 - \delta) \right] \frac{1}{\Delta_k(z, k, b)} = \mathbb{E} \left[\frac{\partial \pi(z', k'(z, k, b))}{\partial k} \middle| z \right]$$

Capital wedges measure the firm-specific distortion in capital allocation, with $\Delta_k = 1$ for unconstrained firms and smaller values indicating greater financial frictions. I summarize the distribution of $1/\Delta_k$ using several moments reported in Table 5.

Let $SD(\log(1/\Delta_k))$ denote the cross-sectional standard deviation of $\log(1/\Delta_k)$, a measure

Figure 1: Effect of Markup Subsidy on Net-worth Cutoff

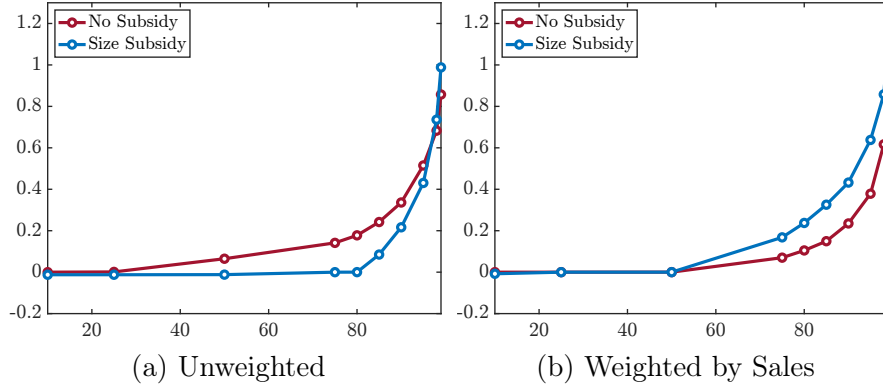


of dispersion in marginal products of capital analogous to the dispersion statistics in [Hsieh and Klenow \(2009\)](#). Let $SD_s(\log(1/\Delta_k))$ denote the sales-weighted analogue, which better captures the impact of financial frictions on aggregate productivity since large firms account for a disproportionate share of output. Skewness is measured using Kelly skewness, which is robust to outliers. Finally, $\text{Corr}(k, \log(1/\Delta_k))$ captures the relationship between firm size and the severity of financial frictions.

Between the baseline and the subsidized economy, both the skewness and unweighted dispersion of $1/\Delta_k$ increase. The gap in dispersion is substantially larger when weighted by sales. For instance, $SD_s(\log(1/\Delta_k))$ rises from 0.150 to 0.239. This indicates that the markup policy disproportionately tightens financial frictions for large firms. The correlation between capital and capital wedges shifts from -0.283 to 0.008 . The implication is that, in the baseline economy, large firms tend to face looser financial frictions, whereas in the economy without markups, this relationship disappears. Large firms, which account for a disproportionate share of output, now face financial frictions comparable to small firms. The ratio $\log(\mathcal{U}/\mathcal{M})$ also increases from 0.084 to 0.107, confirming that the contribution of financial frictions to aggregate misallocation grows when markups are eliminated.

Figure 2 plots the percentiles of $\log(1/\Delta_k)$ with and without the markup policy, both unweighted and weighted by sales. The distribution shifts significantly under the size-dependent subsidy, and the shift is markedly more pronounced when weighted by sales, reflecting the disproportionate impact of financial frictions on large firms when markups are eliminated.

Figure 2: Percentiles of $\log \frac{1}{\Delta}$



3.4 Alternative Policies

The previous section established that eliminating markups worsens financial frictions. Markups distort through two channels: the level of the aggregate markup and the dispersion of firm-level markups. Which channel is responsible for alleviating financial frictions?

To answer this, I generalize the size-dependent subsidy to allow independent control over each channel. Consider the following transfer to a firm with relative output q :

$$T(q) = YD \left[\frac{1}{1+\omega} \frac{\Upsilon(q)^{1+\xi}}{1+\xi} - \Upsilon'(q)q \right] \quad (9)$$

This subsidy implies that prices are

$$\underbrace{\Upsilon'(q)D}_{\text{price}} = (1+\omega)\Upsilon(q)^{-\xi} \underbrace{\frac{1}{q} \frac{WY^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \left(\frac{q}{zk^\alpha} \right)^{\frac{1}{1-\alpha}}}_{\text{marginal cost}} \quad (10)$$

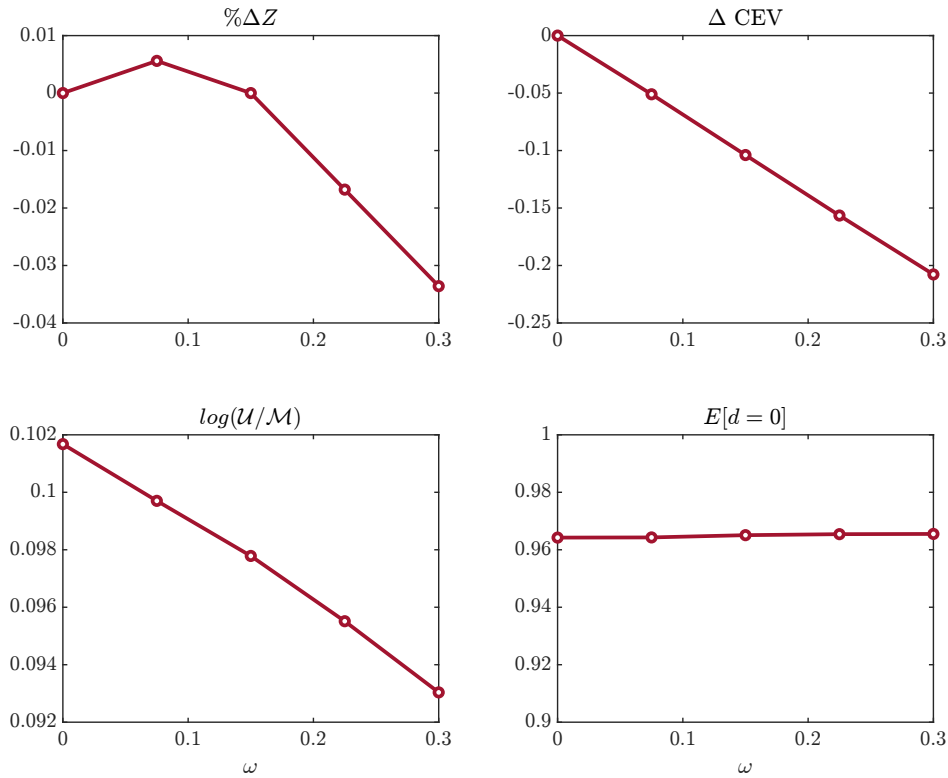
so that firm-level markups equal $(1+\omega)\Upsilon(q)^{-\xi}$. The parameters ω and ξ control the level and dispersion of markups, respectively. Setting $\xi = \omega = 0$ recovers the benchmark policy that eliminates markups entirely. Setting $\xi = 0$ and $\omega = \mathcal{M} - 1$ eliminates markup dispersion while maintaining a common markup equal to $\mathcal{M}$. Setting $\omega = 0$ and $\xi < 0$ allows markup dispersion such that markups increase with firm size, while the aggregate markup is determined by the sales-weighted harmonic average of individual markups.

To gauge how each channel contributes to the results in Table 5, I use the transfers in equation (9) to conduct two exercises.¹³

¹³For completeness, a policy that eliminates the aggregate markup $\mathcal{M}$ via a uniform tax on labor while retaining markup variation delivers 20 percent CEV gains, and is therefore dominated by the policies that follow.

Exercise # 1: Constant Markup. To assess whether the level of the aggregate markup affects financial frictions, I fix $\xi = 0$, eliminating markup dispersion, and vary ω so that all firms share a common markup $1 + \omega$. Figure 3 plots changes in productivity, CEV welfare relative to the no-markup economy, $\log(\mathcal{U}/\mathcal{M})$, and fraction of firms with zero dividends for different values of ω , fixing $\xi = 0$. Aggregate productivity varies non-linearly with ω . However, the magnitudes are very small. Welfare falls monotonically as the aggregate wedge $1 + \omega$ increases from 0 to 1.30. Both the capital wedge and fraction of constrained firms vary very little. Overall, as the aggregate markup varies, the severity of financial frictions remains unchanged. However, increasing the aggregate markup depresses the wages of the representative household, leading to significant welfare losses.

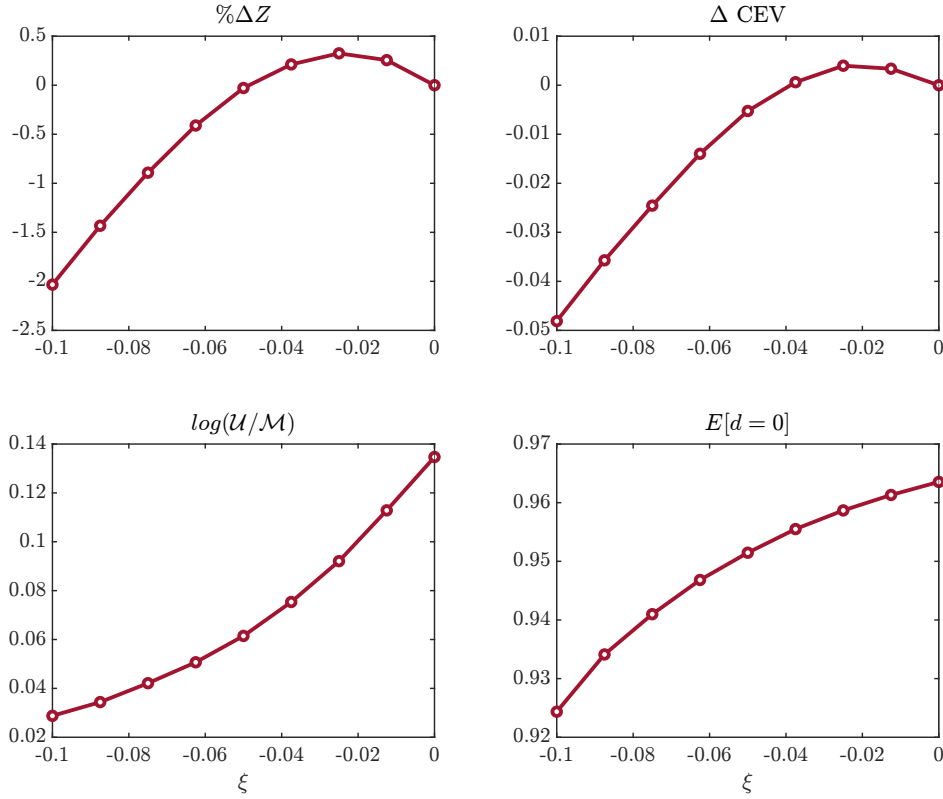
Figure 3: Alternative Policy: Varying the level of $\mathcal{M}$



Exercise # 2: Markup Dispersion. Figure 4 plots the same four outcomes as above but now varies ξ while holding $\omega = 0$. This exercise eliminates the aggregate markup but allows for some markup dispersion. The benchmark policy corresponds to $\xi = 0$. Allowing markup dispersion ($\xi < 0$) generates gains in both productivity and welfare. The reason is apparent in the lower panels: the portion of the capital wedge attributed to financial frictions, $\mathcal{U}/\mathcal{M}$, and the fraction of constrained firms both fall as ξ becomes more negative. In summary,

allowing markups to vary with firm size reduces the severity of financial frictions sufficiently to generate net gains in both productivity and welfare relative to the no-markup benchmark.

Figure 4: Alternative Policy: Allowing for Markup Dispersion



Together, the two exercises establish that markup dispersion, rather than the level of the aggregate markup, is responsible for reducing the severity of financial frictions.

3.5 Optimal Policy

I now solve for the optimal policy within the generalized subsidy schedule introduced in equation (9). I choose the parameters ω and ξ to maximize consumption-equivalent welfare along the transition path from a permanent introduction of the policy.¹⁴ The benchmark for comparison is the policy that eliminates markups entirely, which corresponds to $\xi = \omega = 0$.

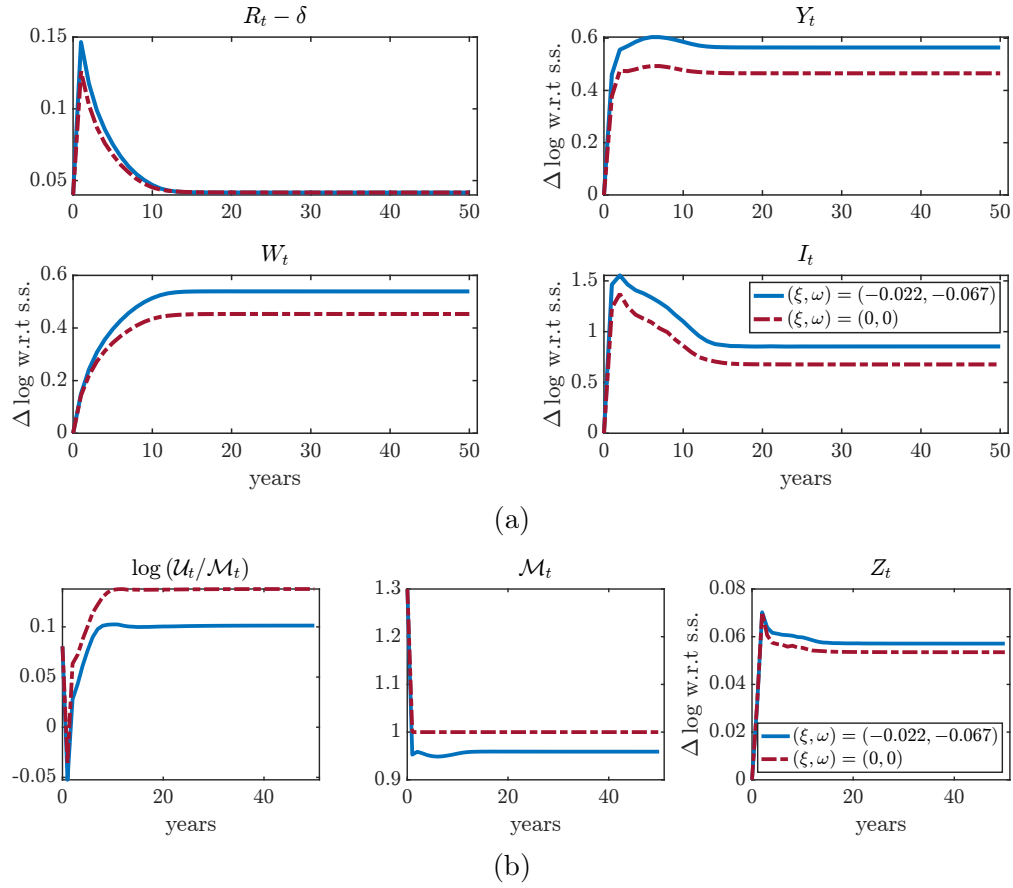
The optimal parameters are $\omega = -0.067$ and $\xi = -0.022$. Welfare gains are 18 percent in consumption-equivalent terms, approximately 1 percent greater than under the policy that

¹⁴CEV for transition paths is Δ where $\frac{\log(C(1+\Delta)) - \Psi L_t}{1-\beta} = \sum_{t \geq 0} \beta^t \left\{ \log(\tilde{C}_t) - \Psi \tilde{L}_t \right\}$, where $\tilde{C}_t$ and $\tilde{L}_t$ are paths under the policy.

eliminates markups. Productivity gains between steady states are 5.7 percent, compared with 5.3 percent under the markup-eliminating policy.

Transition dynamics. Figures 5a and 5b plot the transition paths for the optimal policy $(\xi, \omega) = (-0.022, -0.067)$ and the markup-eliminating policy $(\xi, \omega) = (0, 0)$. Figure 5a plots the interest rate net of depreciation ($R_t - \delta$), output (Y_t), wages (W_t), and investment (I_t), each expressed as log-deviations from the initial steady state. Figure 5b plots aggregate productivity (Z_t), the capital wedge relative to the aggregate markup ($\log(\mathcal{U}_t/\mathcal{M}_t)$), and the aggregate markup ($\mathcal{M}_t$).

Figure 5: Transition Paths of Policy

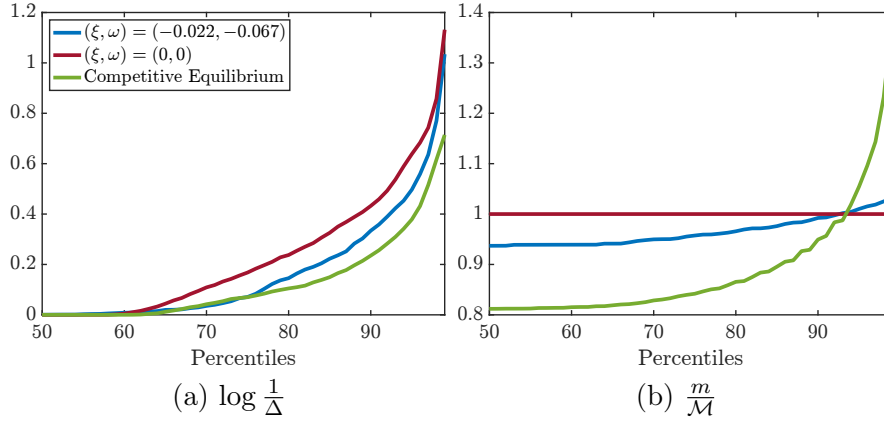


On impact, the policy reduces the aggregate markup significantly. From the perspective of workers, an implicit tax on their wages has fallen, prompting them to increase their supply of labor. The increase in output is larger under the optimal policy than the policy that eliminates markups. In the long run, the capital wedge relative to the markup, $\log(\mathcal{U}_t/\mathcal{M}_t)$, is lower under the optimal policy than under the policy that eliminates variable markups.

Financial frictions are less severe under the optimal policy than under a policy that eliminates all markups.

To interpret the optimal parameters, recall that output decisions in this context solve equation (10). First, the parameter $\xi \neq 0$ generates markup dispersion. In Figures 6a and 6b, I compare the distribution of markups and credit wedges. I plot the sales-weighted percentiles of these wedges for each steady state. To ease interpretation, markups are plotted relative to the aggregate markup $\mathcal{M}$. I compare the competitive equilibrium to the economy under the optimal parameters (ω, ξ) and to the economy under the size-dependent subsidy that eliminates markups entirely, i.e., $\omega = \xi = 0$. Because $\xi \neq 0$, the optimal policy reduces

Figure 6: Wedges in Optimal Policy



the dispersion of credit wedges at the cost of greater dispersion in markups. This confirms the conjecture from Section 3.4 that the optimal policy retains markup dispersion because it alleviates losses from financial frictions. In the next section, I confirm that this result is a core feature of the planner's constrained-efficient allocation.

Second, because $\omega < 0$, the optimal policy includes a significant output subsidy. Fixing $\xi = 0$ and letting $\omega < 0$, firms produce more than in the frictionless economy. In Figure 5b, the aggregate markup in the optimal policy is a *mark-down*.¹⁵ Combining the income shares of labor and capital, the aggregate capital wedge can be expressed as $\mathcal{U}_t = \mathcal{M}_t \frac{\alpha W_t L_t}{(1-\alpha) R_t K_{t+1}}$. All else equal, a lower aggregate markup can also reduce distortions in the allocation of capital.

¹⁵Including transitions in the welfare calculation is important. Without the inclusion of transitions in the welfare calculation, the aggregate markup would no longer be a *mark-down* under the optimal policy but rather significantly greater than one.

4 Constrained Efficiency

The decomposition of productivity losses in equation (8) suggests that markups alleviate losses attributed to financial frictions. The optimal policy, restricted to subsidy schedule (9), suggests that the planner ought to trade off some markup dispersion for improvements in the allocation of capital. In this section, I investigate the constrained-efficient Ramsey planner's allocation in this environment.

I first characterize the constrained planner's problem, then explore the implications quantitatively in a numerically tractable calibration. Unlike laissez-faire allocations, constrained-efficient policies with financial frictions and variable markups directly tie capital investment decisions to contemporaneous output decisions. Firms facing significant financial frictions charge higher markups over marginal cost. Less constrained firms charge lower markups. This leads to a correlation between the marginal product of capital and markups that is more positive under the planner's allocation than in the competitive equilibrium.

4.1 Constrained Planner's Problem

The planner's problem is to maximize the lifetime utility of the representative household

$$S(\mu_0) = \sum_{t=0}^{\infty} \beta^t \{ \log C_t - \Psi L_t \}$$

given an initial distribution μ_0 of firms. The planner is constrained by the law of motion $\mu' = T(\mu)$, which is detailed in equation (14). They choose aggregate quantities $\{Y_t, C_t, L_t\}$, prices $\{W_t, D_t\}$, and firm-specific quantities $\{k'_t(x), b'_t(x), q_t(x)\}$, that is capital, debt, and relative output, respectively, for firms in states $x = (z, k, b)$. The planner makes these choices to replicate the competitive equilibrium, subject to the financial frictions in the environment. A detailed list of constraints and all derivations are available in Appendix A.6. To simplify the exposition, I fix the price of debt at its steady-state value, $q_b = \beta$, and restrict attention to the steady-state allocation. Pecuniary externalities from interest rates are not the focus of this paper. In addition, to simplify notation I suppress the firm exit probability since it is not material for the results that follow.

Define $\lambda_d(z, k, b)$ and $\lambda_b(z, k, b)$ as the firm-specific multipliers on the dividend and borrowing constraints. Let $\tilde{\lambda}_d = \frac{\lambda_d}{U_c}$ be the dividend constraint multiplier in marginal utility of consumption terms. Define other multipliers similarly. Finally, define $\lambda_f = \lambda_d + \phi \lambda_b$, a term reflecting the total cost of financial frictions of a particular firm from the social planner's perspective. Define $E_x(y) = \int \frac{x_i}{\int x_i di} y_i di$ as the expectation of variable y weighted by x .

4.2 Capital Choice

Proposition 2. *The social planner's capital choice $k'(x)$ for a firm in state $x = (z, k, b)$ is such that*

$$(1 - \Lambda_W \Psi)(1 - \beta(1 - \delta)) + \tilde{\lambda}_d(x) = \beta \left\{ E \left[\tilde{\lambda}_d(x') | z \right] + E \left[\left(1 + \tilde{\lambda}_d(x') + \tilde{\lambda}_b(x') \phi \right) \frac{\partial \pi(z', k'(x))}{\partial k} | z \right] \right\}$$

where $\Lambda_W = LE_\ell(\tilde{\lambda}_f)$ is the multiplier on the representative household's labor decision

Proposition 2 details the capital choice of the planner. This condition resembles its competitive equilibrium counterpart, with one important difference. The planner internalizes the effect of investment choices of firms on wages. This occurs because the household's labor decision is determined by $W = \Psi C$. Changes in investment, all else equal, reduce consumption and hence wages. The income effect on labor supply in preferences contributes to this channel. This term would not be present if households had Greenwood et al. (1988) preferences. With the income effect, capital is subsidized in the constrained-efficient allocation relative to competitive equilibrium so that wages fall. The multiplier faced by the planner on the household's labor decisions, Λ_W , can be expressed as the labor-weighted average cost of financial frictions. If financial frictions are not costly, the motive to subsidize capital investment disappears.

4.3 Markups

Proposition 3. *The social planner's output choice for a firm in state $x = (z, k, b)$ implies that $m(x)$ is*

$$\frac{1}{m(x)} = \frac{1}{1 + \tilde{\lambda}_f(x)} \frac{1}{m^{unc}(x)} + \frac{\tilde{\lambda}_f(x)}{1 + \tilde{\lambda}_f(x)} \frac{\theta(x) - 1}{\theta(x)}$$

where

$$\frac{1}{m^{unc}(x)} = 1 + \Lambda_K - \Lambda_D \frac{\theta(x) - 1}{\theta(x)}$$

and Λ_K and Λ_D are the multipliers on the Kimball aggregator and demand shifter D .

Proposition 3 describes the markups that the constrained social planner chooses for a firm. The firm-specific markup is a harmonic average of unconstrained firm markups (m^{unc}) and markups in the competitive equilibrium, with weights that are functions of the cost of

financial frictions, λ_f . As the firm becomes very financially constrained, markups approach the level in the competitive equilibrium where markups are described by the Lerner index, $\frac{\theta(q)}{\theta(q)-1}$, where $\theta(q)$ is the elasticity of demand.

Markups and Financial Frictions. Why do financial frictions show up in markups? For simplicity, suppose $\Lambda_D = \Lambda_K = 0$, setting aside the macro externalities to be discussed below. The first-order condition for q is then

$$(1 + \tilde{\lambda}_f) MC(q) = p(q) \left\{ 1 + \tilde{\lambda}_f \left(1 + \frac{\partial \log q}{\partial \log p} \right) \right\} \quad (11)$$

That is, marginal costs are equated with marginal benefit. The "physical" marginal cost of q is the labor hired, here denoted $MC(q)$ for brevity. However, hiring more labor tightens the dividend and borrowing constraints, at an additional cost of $\tilde{\lambda}_f MC(q)$. With respect to marginal benefit, variation in q generates $p(q) \propto \Upsilon'(q)$. Analogously, $p(q)$ increases revenues and relaxes the financial constraints. Changes in q increase revenues by $p(q)$ and by changing the elasticity of demand, $\frac{\partial \log q}{\partial \log p}$.

If financial frictions do not matter ($\tilde{\lambda}_f = 0$), then the firm does not charge a markup. If financial frictions are severe ($\tilde{\lambda}_f > 0$), then markups approach the familiar Lerner index from competitive equilibrium. The positive interaction between markups and financial frictions is therefore quite clear. When financial frictions are severe, their costs can be mitigated through higher profits. The planner can generate these profits only by allowing firms to price above marginal cost.

Unconstrained Firms. Financially unconstrained firms set prices that internalize shortfalls in output due to constrained firms and pecuniary externalities from strategic complements. The externalities are present through the multipliers on the Kimball aggregator (Λ_K) and demand shifter (Λ_D). The externality on the Kimball aggregator can be expressed as

$$\Lambda_K = \int \tilde{\lambda}_f(x) \{ \Upsilon'(q(x))q(x) - \Psi\ell(x) \} d\mu(x) - Cov_s \left(\tilde{\lambda}_f, \frac{\theta - 1}{\theta} \right)$$

The first term is the surplus generated by a constrained firm, the marginal revenue net of utility cost of their labor demand. It is large if constrained firms are responsible for a significant share of sales. In such a case, the markup of the unconstrained firm falls. Potentially, as in the restricted optimal policy exercise in Section 3.5, they produce with a *mark-down* to help divert output away from constrained to unconstrained firms.

The second term reflects motives to divert output from constrained to unconstrained

firms depending on the strength of the market power and financial frictions interaction. It is the covariance between the cost of financial frictions and the inverse of markups in the competitive equilibrium. If financial frictions are associated with large firms, for which $\frac{\theta(q)-1}{\theta(q)}$ is high, then Λ_K is larger and output choice is higher. From equation (11), the coincidence of financial costs and competitive equilibrium markups determines output. If large firms are financially constrained, then the constrained-efficient planner wants to give them a markup to alleviate financial frictions. As the loss of their output is significant, the planner increases the production of unconstrained firms, potentially producing with a *mark-down*.

Returning to $m^{unc}(x)$, the final term is the firm-specific contribution to the demand shifter. This term corrects for a pecuniary externality. The planner internalizes a firm's contribution to the demand shifter, which in this environment reflects strategic complements. Λ_D can be expressed as:

$$\Lambda_D = E_s \left(\tilde{\lambda}_f \right)$$

that is, the sales-weighted average of the shadow cost of financial frictions. Small changes in D expand revenues and relax the dividend and borrowing constraints for all constrained firms. Hence, the strength of this externality is in proportion to the sales-weighted average cost of financial frictions.

To summarize, Proposition 3 suggests that the planner allocates higher profits to severely constrained firms to promote investment, while ensuring that unconstrained firms compensate for the resulting shortfall in output. As illustrated by restricted optimal policy in Section 3.5, markups are not necessarily eliminated. Instead, their variation critically depends on financial frictions.

Aggregate Markups. Proposition 3 refers to firm-specific markups. For the aggregate markup, the average degree of financial frictions will be relevant. Equation (12) expresses the aggregate markup in the constrained-efficient environment

$$\mathcal{M} = \frac{1 + E_\ell \left(\tilde{\lambda}_f \right)}{1 - \Psi L E_\ell \left(\tilde{\lambda}_f \right) + E_s \left(\tilde{\lambda}_f \right)} \quad (12)$$

Crucially, the average social cost of the financial frictions shifts the aggregate markup. If financial frictions are not present then $\mathcal{M} = 1$. If some constrained firms are present but they are responsible for a very small share of sales, $E_s \left(\tilde{\lambda}_f \right)$ is small, or demand a very small share of labor, $E_\ell \left(\tilde{\lambda}_f \right)$ is small, then the aggregate markup is approximately one.

Constant Elasticity of Substitution. The model nests CES demand if the super-elasticity of demand $\varepsilon = 0$, making $\theta(q) = \sigma$. In such a case $D = 1$ and $\mathbf{\Lambda}_D = 0$. However, constrained-efficient allocations can feature variable markups even though markups are constant in the competitive equilibrium. Markups will vary if the costs of financial frictions vary.

Exogenous Capital Wedges. Unlike the case of CES, where markups are constant in competitive equilibrium, if wedges in the capital choice of firms were exogenous, then constrained-efficient policy would eliminate markups. The crucial interaction between markups and financial frictions arises because markups affect current profits and thereby alter capital demand.

4.4 Implementation

Proposition 4. *The social value function, $V^p(x) \equiv \frac{1}{U_c} \frac{\partial S}{\partial \mu(x)}$, of a firm in state $x = (z, k, b)$ is*

$$\begin{aligned} V^p(x) &= \max_{b', k'} \text{div} + \beta E(V^p(x')|z) \\ \text{s.t.} \quad &\text{div} = \pi(z, k) + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c + T^P(x) \\ &b' \leq \phi(\pi(z, k) + T^P(x)) \\ &\text{div} \geq 0 \end{aligned}$$

where $T^P(x)$ are state dependent transfers defined as

$$\begin{aligned} T^P(x) &= \frac{1}{1 + \tilde{\lambda}_f(x)} DY \{ \xi_0 \Upsilon(q(x)) - \xi_1 \Upsilon'(q(x))q(x) \} + \frac{\Psi L E_\ell(\tilde{\lambda}_f)}{1 + \tilde{\lambda}_f(x)} (k'(x) - (1 - \delta)k) \\ \xi_0 &= 1 + \mathbf{\Lambda}_K \\ \xi_1 &= 1 + \mathbf{\Lambda}_D \end{aligned}$$

Proposition 4 describes the social value function of a particular firm. Solutions to this Bellman equation implement the social planner's allocation. If the social cost of financial frictions $\tilde{\lambda}_f$ were known in every state, and the transfers $T^P(x)$ were constructed accordingly, private firms would choose allocations consistent with the constrained-efficient allocation.

I can rearrange this transfer into a more recognizable decomposition:

$$\begin{aligned}
T^P(x) = & \quad DY \{ \Upsilon(q(x)) - \Upsilon'(q(x))q(x) \} \quad (\text{Markups}) \\
& - \frac{\tilde{\lambda}_f(x)}{1 + \tilde{\lambda}_f(x)} DY \{ \Upsilon(q(x)) - \Upsilon'(q(x))q(x) \} \quad (\text{Financial Frictions}) \\
& + \frac{1}{1 + \tilde{\lambda}_f(x)} DY \{ \Lambda_K \Upsilon(q(x)) - \Lambda_D \Upsilon'(q(x))q(x) \} \quad (\text{Pecuniary Externalities}) \\
& + \frac{\Psi LE_\ell(\tilde{\lambda}_f)}{1 + \tilde{\lambda}_f(x)} (k'(x) - (1 - \delta)k) \quad (\text{Capital Subsidy}) \quad (13)
\end{aligned}$$

There are four components: (1) a transfer to counteract market power, (2) adjustments to the transfer if firms are constrained, (3) adjustments to the transfer to internalize pecuniary externalities, and (4) capital subsidy to internalize the effect of investment on labor supply. Observe that the final two components, pecuniary externalities and capital subsidy, are higher for unconstrained firms, all else equal. Therefore, the constrained planner makes it the responsibility of unconstrained firms to internalize these two distortions.

Compare these transfers to ones introduced earlier to eliminate markups

$$T(x) = DY [\Upsilon(q(x)) - \Upsilon'(q(x))q(x)]$$

They only share the first term, which counteracts the variable markups. To make progress on understanding the inefficiency of policy that eliminates markups, I restrict comparison of the two transfers to the first two terms of T^P in equation (13). The reason is that the size of the pecuniary externality is difficult to ascertain analytically and the capital subsidy can be small if Ψ is small. I focus instead on the first two terms, which reflect the planner's desire to counteract market power while accommodating financial frictions. With these caveats in mind, the difference between the two transfers, for a firm with the same output choices under both policies, is:

$$T(x) - T^P(x) = \frac{\tilde{\lambda}_f(x)}{1 + \tilde{\lambda}_f(x)} DY \{ \Upsilon(q(x)) - \Upsilon'(q(x))q(x) \}$$

If the social cost of financial frictions is very large, such that $\frac{\tilde{\lambda}_f(x)}{1 + \tilde{\lambda}_f(x)} \rightarrow 1$, then $T^P \rightarrow 0$. Therefore, a policy that eliminates markups transfers too much to constrained firms. On the other hand, if a firm's financial frictions are not socially costly, then the policy to eliminate markups is the correct size.

4.5 Illustrative Example

In this section, I present results from a limited model. In the limited model, I change worker preferences to $U(C, L) = C - L$. This means that $W = 1$ in competitive equilibrium. Firm productivity is made permanent ($\rho_z = 1$), firms only live for two periods, $\pi_d = 1$, and capital fully depreciates, $\delta = 1$.¹⁶ As in the main model, firms enter with (z_0, k_0, b_0) where z_0 is drawn from the Pareto distribution with shape $\zeta > 0$, and initial debt and capital remain as in the benchmark calibration. Firms solve identical problems, subject to the same financial frictions and demand for their products. In the second period, they produce, pay back their debt, and exit. One can think of firms in the first and second periods as “young” and “old” firms, respectively.

The benefit of this limited calibration is that the cost of financial friction, $1 + \lambda_f$, is exactly equal to the marginal product of capital (MPK). Hence, the weights in Proposition 3 map directly to the MPK. Rents from production are allocated in proportion to the firm’s implicit cost of capital. To see this more clearly, compare markups when the planner is in control versus laissez-faire:

$$\frac{1}{m(q)} - \frac{\theta(q) - 1}{\theta(q)} = \frac{1}{\beta \frac{\partial \pi(z, k')}{\partial k}} \left\{ 1 + \Lambda_K - (1 + \Lambda_D) \frac{\theta(q) - 1}{\theta(q)} \right\}$$

All else equal, higher MPK firms ought to have higher markups. In the competitive equilibrium, this trade-off is not guaranteed due to Marshall’s Second Law of Demand, where markups increase in firm size. There are two reasons for this particular positive interaction between markups and financial frictions in the planner’s allocation: (1) Giving higher markups to constrained firms is equivalent to reallocating output from constrained to unconstrained firms, thus reducing aggregate distortions through the aggregate capital wedge $\mathcal{U}$. (2) Increasing markups increases profits and thus reduces capital distortions. Both motives are present and are difficult to disentangle.

In this illustrative environment,

$$\Lambda_D = E_s \left(\beta \frac{\partial \pi(z, k')}{\partial k} \right) - 1, \quad \Lambda_K = E_s \left(\beta \frac{\partial \pi(z, k')}{\partial k} \right) - Cov_s \left(\beta \frac{\partial \pi(z, k')}{\partial k}, \frac{\theta - 1}{\theta} \right) - 1$$

Since returns to capital are $\beta \frac{\partial \pi(z, k')}{\partial k} - 1$, externalities through output and through strategic complements increase as the returns to capital increase. Within the context of implementable transfer policy (13), average marginal returns, weighted by sales, determine the average sales and output subsidy. The more misaligned returns to capital are across firms, on average,

¹⁶These restrictions, made for illustration, are similar to [Lanteri and Rampini \(2023\)](#).

the greater the need for output to be subsidized uniformly. This is captured well by the restricted optimal policy in Section 3.5.

Given the simplicity of this limited environment, I can solve numerically for policies and compare them, across steady states, to the size-dependent subsidy which eliminates markups. Table 6 compares productivity (Z), aggregate markup ($\mathcal{M}$), aggregate capital wedge $\mathcal{U}$, and the percentage of constrained firms, relative to the competitive equilibrium. I report the two externalities Λ_K and Λ_D , which are only present in the constrained-efficient allocation.

Table 6: Constrained-Efficient Allocation

	Baseline	Eliminate Markups	Constrained Efficient
Z	2.010	2.189	2.193
$\mathcal{M}$	1.275	1.000	1.026
$\mathcal{U}$	0.363	0.770	0.735
% Constrained	0	17	14
Λ_D			0.073
Λ_K			0.029

As in the policy exercise of Section 3.5, allowing some markup variation improves the allocation of capital. This leads to higher aggregate productivity. The first panel, Figure 7a, compares markup percentiles in the competitive equilibrium and the constrained-efficient allocation. In Figure 7b capital wedges are compared between policies to eliminate markups and constrained-efficient allocations. Overall, the trade-off between markups and financial frictions is present in the constrained-efficient allocation.

Unlike the competitive equilibrium, markups are no longer guaranteed to increase in size. Figure 8 plots the capital wedge λ_f and markups $m(q)$ for young firms by productivity z . For unconstrained firms, markups are decreasing in z . They begin increasing for constrained firms, in proportion to their marginal product of capital.

Markup policies for the constrained-efficient allocation are dependent on the initial distribution of wealth among firms as well as primitive parameters governing preferences. For instance, with a super-elasticity of $\varepsilon/\sigma = 0.16$ as in Edmond et al. (2023), optimal markups can be increasing in size for young firms. Figure 9 plots the analogue to Figure 8 with this change in super-elasticity of demand.

Figure 7: Percentiles of Wedges

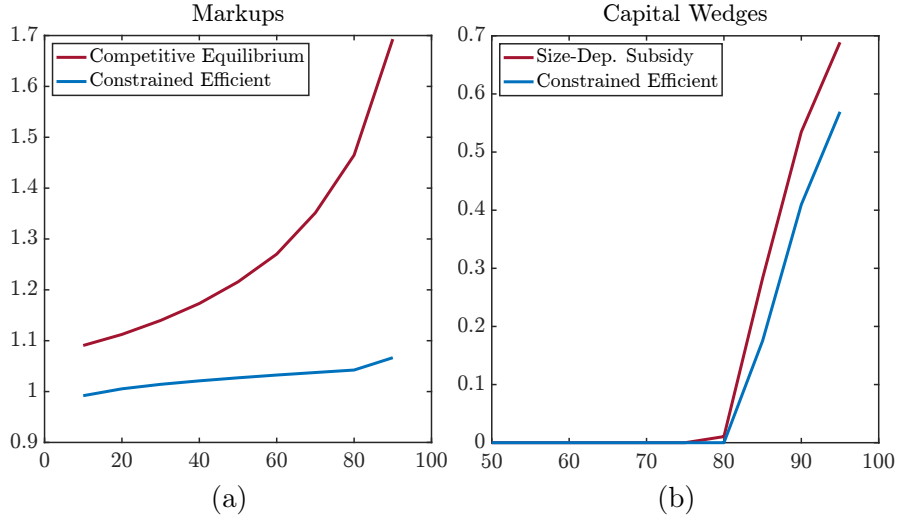
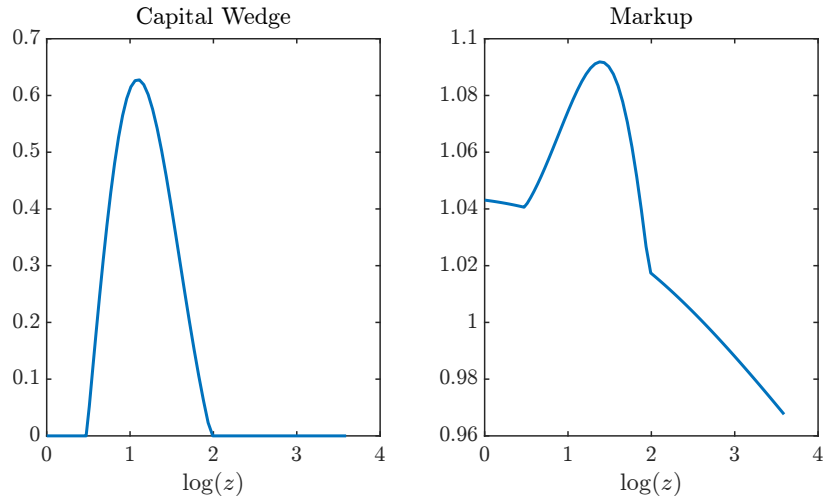


Figure 8: Wedges in Optimal Policy

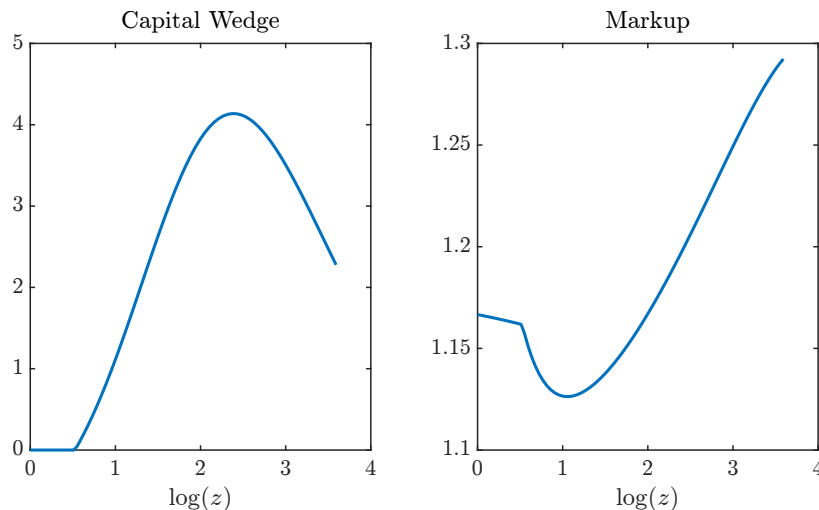


5 Conclusion

How should policies targeting market power account for financial frictions? This paper studies optimal product market intervention when firms face both variable markups and financial frictions. In a quantitative model calibrated to U.S. non-financial corporations, I decompose total misallocation into contributions from each distortion and their interaction. The interaction is quantitatively important, accounting for approximately 25 percent of total productivity losses. I attribute this interaction to the tightening of financial frictions that occurs when markups are eliminated.

The analysis yields two main findings. First, the dispersion of markups, rather than

Figure 9: Wedges in Optimal Policy: Alternative Super-Elasticity ($\varepsilon/\sigma = 0.16$)



the level of the aggregate markup, is responsible for alleviating financial frictions when firms have market power. This finding suggests that policies designed to curb market power should account for the role of markups in relaxing financial constraints. Second, in the constrained-efficient allocation, each firm's markup is a harmonic average of the unconstrained optimum and the competitive equilibrium markup, weighted by the shadow cost of its financial frictions. As a firm's financing costs increase, its markup approaches the competitive equilibrium level. A policy that eliminates markups therefore transfers too much to precisely the firms for which markups are most valuable, yielding lower welfare than the constrained-efficient allocation.

Several directions for future research emerge from this analysis. First, endogenizing entry and the degree of competition in the presence of financial frictions is a natural extension, as the welfare consequences of market power may differ substantially when entry itself is subject to financial frictions. Second, the analysis abstracts from aggregate uncertainty. Business cycle fluctuations may amplify or dampen the interaction between markups and financial frictions, particularly if markups are procyclical and financial constraints bind more tightly in downturns. Third, financial frictions are modeled as exogenous borrowing constraints. Endogenizing the lending relationship through optimal contracting between banks and firms would allow one to examine whether borrowers' market power affects the terms of credit and, in turn, the equilibrium distribution of markups.

References

- Amiti, M., Itskhoki, O., and Konings, J. (2019). International shocks, variable markups, and domestic prices. *The Review of Economic Studies*, 86(6):2356–2402.
- Baqae, D. R. and Farhi, E. (2020). Productivity and misallocation in general equilibrium. *The Quarterly Journal of Economics*, 135(1):105–163.
- Boar, C. and Midrigan, V. (2023). Markups and inequality.
- Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and development: A tale of two sectors. *American economic review*, 101(5):1964–2002.
- Buera, F. J. and Shin, Y. (2013). Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy*, 121(2):221–272.
- Buera, F. J. and Shin, Y. (2017). Productivity growth and capital flows: The dynamics of reforms. *American Economic Journal: Macroeconomics*, 9(3):147–185.
- Crouzet, N. and Mehrotra, N. R. (2020). Small and large firms over the business cycle. *American Economic Review*, 110(11):3549–3601.
- Dávila, E. and Korinek, A. (2018). Pecuniary externalities in economies with financial frictions. *The Review of Economic Studies*, 85(1):352–395.
- De Loecker, J., Eeckhout, J., and Unger, G. (2020). The rise of market power and the macroeconomic implications. *The Quarterly Journal of Economics*, 135(2):561–644.
- Demirer, M. (2022). Production function estimation with factor-augmenting technology: An application to markups.
- Drechsel, T. (2023). Earnings-based borrowing constraints and macroeconomic fluctuations. *American Economic Journal: Macroeconomics*, 15(2):1–34.
- Edmond, C., Midrigan, V., and Xu, D. Y. (2023). How costly are markups? *Journal of Political Economy*, 131(7):000–000.
- Gopinath, G., Kalemli-Özcan, Ş., Karabarbounis, L., and Villegas-Sanchez, C. (2017). Capital allocation and productivity in South Europe. *The Quarterly Journal of Economics*, 132(4):1915–1967.
- Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417.

- Guner, N., Ventura, G., and Xu, Y. (2008). Macroeconomic implications of size-dependent policies. *Review of economic Dynamics*, 11(4):721–744.
- Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing tfp in china and india. *The Quarterly journal of economics*, 124(4):1403–1448.
- Itskhoki, O. and Moll, B. (2019). Optimal development policies with financial frictions. *Econometrica*, 87(1):139–173.
- Jo, I. H. and Senga, T. (2019). Aggregate consequences of credit subsidy policies: Firm dynamics and misallocation. *Review of Economic Dynamics*, 32:68–93.
- Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.
- Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model.
- Klenow, P. J. and Willis, J. L. (2016). Real rigidities and nominal price changes. *Economica*, 83(331):443–472.
- Lanteri, A. and Rampini, A. A. (2023). Constrained-efficient capital reallocation. *American Economic Review*, 113(2):354–395.
- Li, H., Lian, C., Ma, Y., and Martell, E. (2025). Borrowing constraints, markups, and misallocation.
- Lian, C. and Ma, Y. (2021). Anatomy of corporate borrowing constraints. *The Quarterly Journal of Economics*, 136(1):229–291.
- Midrigan, V. and Xu, D. Y. (2014). Finance and misallocation: Evidence from plant-level data. *American economic review*, 104(2):422–458.
- Ottonello, P., Perez, D. J., and Varraso, P. (2022). Are collateral-constraint models ready for macroprudential policy design? *Journal of International Economics*, 139:103650.
- Ottonello, P. and Winberry, T. (2020). Financial heterogeneity and the investment channel of monetary policy. *Econometrica*, 88(6):2473–2502.
- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.

A Derivations

A.1 Distribution

Let $\mu(z, k, b)$ be the distribution of firms actively producing across states (z, k, b) . It is consistent with respect to the following law of motion

$$\begin{aligned} \mu(\mathcal{Z} \times \mathcal{K} \times \mathcal{B}) &= (1 - \pi_d) \int \mathbb{1}(z \in \mathcal{Z}) \mathbb{1}(k \in \mathcal{K}) \mathbb{1}(b \in \mathcal{B}) d\hat{\mu}(z, k, b) + \\ &\quad \pi_d \int \mathbb{1}(z \in \mathcal{Z}) \mathbb{1}(k_0 \in \mathcal{K}) \mathbb{1}(b_0 \in \mathcal{B}) dF_0(z) \end{aligned} \quad (14)$$

where $\mathcal{Z} \times \mathcal{K} \times \mathcal{B}$ is a set belonging to the product σ -algebra of $\mathbb{Z} \times \mathbb{K} \times \mathbb{B}$, the Cartesian product for the supports of productivity, capital, and debt, respectively. Measure $\hat{\mu}$ is of incumbent firms which evolves according to

$$\hat{\mu}(\mathcal{Z} \times \mathcal{K} \times \mathcal{B}) = (1 - \pi_d) \int Q(\mathcal{Z} \times \mathcal{K} \times \mathcal{B} | z, k, b) d\hat{\mu}(z, k, b) + \pi_d \int Q(\mathcal{Z} \times \mathcal{K} \times \mathcal{B} | z, k_0, b_0) dF_0(z)$$

The transition process Q is defined as

$$Q(\mathcal{Z} \times \mathcal{K} \times \mathcal{B} | z, k, b) = \mathbb{1}(k'(z, k, b) \in \mathcal{K}) \mathbb{1}(b'(z, k, b) \in \mathcal{B}) \int_{z' \in \mathcal{Z}} Q_z(z' | z) dz'$$

with Q_z as the transition probabilities associated with the process for z described in equation (1).

A.2 Equilibrium

Definition 1. A stationary equilibrium is a set of prices $\{q_b, W, D\}$, household's allocations $\{C, L\}$, firm policies $\{k'(z, k, b), b'(z, k, b), q(z, k)\}$, and distribution $\mu(z, k, b)$ such that

- Given W and q_b , C and L are optimal for the household
- Given q_b, W, D , $k'(z, k, b)$, $b'(z, k, b)$, and $q(z, k)$ are optimal for firms
- Goods Market Clears: $C = Y - I - f_c$, where $I \equiv (1 - \pi_d) \int k'(z, k, b) d\mu(z, k, b) + \pi_d k_0 - (1 - \delta) \int k d\mu(z, k, b)$
- Kimball Aggregator is satisfied: $\int \Upsilon(q(z, k)) d\mu(z, k) = 1$
- Distribution $\mu(z, k, b)$ satisfies law of motion (14) and $\mu(z, k) = \int \mu(z, k, b) db$

A.3 Firm Policies

The first-order condition of the Bellman (2) for debt is

$$0 = (1 + \lambda_d(z, k, b))q_b - \lambda_b(z, k, b)q_b - \beta(1 - \pi_d)E[(1 + \lambda_d(z', k', b'))|z] - \beta\pi_d$$

where λ_d and λ_b are the Lagrange multipliers for the dividend and borrowing constraint, respectively. Rewriting this condition in sequence form

$$\begin{aligned} 0 &= (1 + \lambda_{d,t})q_b - \lambda_{b,t}q_b - \beta(1 - \pi_d)\mathbb{E}[(1 + \lambda_{d,t+1})|z] - \beta\pi_d \\ 0 &= \lambda_{d,t}q_b - \lambda_{b,t}q_b - \beta(1 - \pi_d)\mathbb{E}[\lambda_{d,t+1}|z] \\ \lambda_{d,t}q_b &= \lambda_{b,t}q_b + \beta(1 - \pi_d)\mathbb{E}[\lambda_{d,t+1}|z] \\ \lambda_{d,t} &= (1 - \pi_d)\mathbb{E}\left[\sum_{i=0}^{\infty} \lambda_{b,t+i}\right] \end{aligned}$$

this result states that the shadow value of the dividend constraint is the present discounted value of the borrowing constraint's. Therefore, $d = 0$ if with positive probability today and/or some time in the future the firm's borrowing constraint binds.

The first-order condition for capital

$$\begin{aligned} (1 + \lambda_d(z, k, b)) &= \beta(1 - \pi_d)E\left[(1 + \lambda_d(z', k', b') + \lambda_b(z', k', b')\phi)\frac{\partial\pi(z', k')}{\partial k}\Big|z\right] + \\ &\quad \beta(1 - \pi_d)E[(1 + \lambda_d(z', k', b'))\{(1 - \delta)\}|z] + \beta\pi_d E\left[\frac{\partial\pi(z', k')}{\partial k} + (1 - \delta)\Big|z\right] \end{aligned}$$

If a firm is unconstrained, i.e. $\lambda_d = 0$ and $\lambda_b = 0$ forever, then the capital choice solves the following

$$1 - \beta(1 - \delta) = \beta\mathbb{E}\left[\frac{\partial\pi(z', k^*(z))}{\partial k}\Big|z\right]$$

Note that

$$\frac{\partial\pi(z, k)}{\partial k} = \frac{\alpha}{1 - \alpha}WY^{\frac{1}{(1-\alpha)\eta}}\left(\frac{q(z, k)}{z}\right)^{\frac{1}{(1-\alpha)\eta}}k^{\frac{1}{\alpha-1}}$$

It follows that

$$k^*(z) = \left\{ \frac{\beta^{\frac{\alpha}{1-\alpha}} W Y^{\frac{1}{(1-\alpha)\eta}} \mathbb{E} \left[\left(\frac{q(z', k^*(z))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \middle| z \right]}{1 - \beta(1 - \delta)} \right\}^{1-\alpha}$$

If a firm is unconstrained, they follow $b^*(z)$ defined in equation 4. The dividends are

$$d^*(z, k, b) = \pi(z, k) - b + (1 - \delta)k - f_c + q_b b^*(z) - k^*(z)$$

This is feasible if $d^*(z, k, b) \geq 0$. Otherwise, the unconstrained capital policy is not feasible. To summarize, for a firm with net worth $n(z, k, b) = \pi(z, k) - b + (1 - \delta)k - f_c$, they are constrained if $n(z, k, b) < k^*(z) - q_b k^*(z)$ and unconstrained otherwise. If a firm is constrained then dividends are zero $d = 0$ and they solve the Bellman with the dividend constraint binding.

A.4 Aggregation

A.4.1 Markup

Define the aggregate markup implicitly

$$\frac{WL}{Y} = \frac{(1 - \alpha)\eta}{\mathcal{M}}$$

The firm-specific labor share

$$\begin{aligned} \frac{W\ell}{py} &= \frac{WY^{\frac{1}{(1-\alpha)\eta}-1}}{pq} \left(\frac{q}{zk^{\alpha\eta}} \right)^{\frac{1}{(1-\alpha)\eta}} \\ &= \frac{(1 - \alpha)\eta}{m(q)} \quad (\text{Sub in optimal price}) \end{aligned}$$

Combine the aggregate and firm-specific labor share

$$\begin{aligned} \frac{W\ell}{py} \frac{Y}{WL} &= \frac{(1 - \alpha)\eta}{m(q)} \frac{\mathcal{M}}{(1 - \alpha)\eta} \\ \frac{\ell}{L} \frac{L}{py} &= \frac{\mathcal{M}}{m(q)} \end{aligned}$$

Integrating across firms we have two expressings for aggregate markup

$$\mathcal{M} = \int \frac{\ell(q_i)}{L} m(q_i) di$$

$$\mathcal{M} = \left[\int \frac{p(q_i)q_i}{m(q_i)} di \right]^{-1}$$

A.4.2 Wedges

Recall

$$\begin{aligned} \frac{\partial \pi(z, k)}{\partial k} &= \frac{\alpha}{1-\alpha} W Y^{\frac{1}{(1-\alpha)\eta}} \left(\frac{q(z, k)}{z} \right)^{\frac{1}{(1-\alpha)\eta}} k^{\frac{1}{\alpha-1}} \\ &= \bar{\pi}(z, k) k^{\frac{1}{\alpha-1}} \end{aligned}$$

Re-writing first-order condition

$$\begin{aligned} (1 + \lambda_d(z, k, b)) &= \beta(1 - \pi_d) E \left[(1 + \lambda_d(z', k', b') + \lambda_b(z', k', b') \phi) \bar{\pi}(z', k') k'^{\frac{1}{\alpha-1}} | z \right] + \\ &\quad \beta(1 - \pi_d) E [(1 + \lambda_d(z', k', b')) (1 - \delta) | z] + \\ &\quad \beta \pi_d E \left[\bar{\pi}(z', k') k'^{\frac{1}{\alpha-1}} + (1 - \delta) | z \right] \end{aligned}$$

Collect $k^{\frac{1}{\alpha-1}}$ and to clean up let $\lambda'_d \equiv \lambda_d(z', k', b')$ and similarly the borrowing multiplier

$$(1 + \lambda_d) - \beta(1 - \pi_d) E [(1 + \lambda'_d)] (1 - \delta) - \beta \pi_d (1 - \delta) = \beta(1 - \pi_d) E \left[\left(\frac{1}{1 - \pi_d} + \lambda'_d + \lambda'_b \phi \right) \bar{\pi}(z', k') k'^{\frac{1}{\alpha-1}} | z \right]$$

Put capital to one side and express as a function of marginal products and multipliers

$$k'^{\frac{1}{1-\alpha}} = \frac{\beta(1 - \pi_d) E_z \left[\left(\frac{1}{1 - \pi_d} + \lambda'_d + \lambda'_b \phi \right) \bar{\pi}(z', k') \right]}{(1 + \lambda_d) - \beta(1 - \pi_d) E [(1 + \lambda'_d)] (1 - \delta) - \beta \pi_d (1 - \delta)}$$

Factor out wage and output

$$k' = \left(\frac{\alpha}{1 - \alpha} W \right)^{1-\alpha} Y^{\frac{1}{\eta}} \left\{ \frac{\beta(1 - \pi_d) E_z \left[\left(\frac{1}{1 - \pi_d} + \lambda'_d + \lambda'_b \phi \right) \left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \right]}{(1 + \lambda_d) - \beta(1 - \pi_d) E [(1 + \lambda'_d)] (1 - \delta) - \beta \pi_d (1 - \delta)} \right\}^{1-\alpha}$$

Define

$$M_k(z, k, b) = \frac{\beta(1 - \pi_d)E_z \left[\left(\frac{1}{1 - \pi_d} + \lambda'_d + \lambda'_b \phi \right) \left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1 - \alpha)\eta}} \right]}{(1 + \lambda_d) - \beta(1 - \pi_d)E[(1 + \lambda'_d)](1 - \delta) - \beta\pi_d(1 - \delta)}$$

It follows that

$$k' = \left(\frac{\alpha}{1 - \alpha} W \right)^{1 - \alpha} Y^{\frac{1}{\eta}} M_k(z, k, b)^{1 - \alpha}$$

We can express capital policy relative to the unconstrained policy

$$M_k(z, k, b) = \frac{\beta}{1 - \beta(1 - \delta)} \mathbb{E} \left[\left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1 - \alpha)\eta}} \right] \Delta_{mc}(z, k, b) (1 + \Delta_{mb}(z, k, b))$$

where Δ_{mc} and Δ_{mb} are wedges in the marginal cost and marginal benefit of capital investment that are expressed below.

$$\Delta_{mc}(z, k, b) = \frac{1 - \beta(1 - \delta)}{\beta} \frac{1}{(1 + \lambda_d) - \beta(1 - \pi_d)E[(1 + \lambda'_d)](1 - \delta) - \beta\pi_d(1 - \delta)}$$

$$\Delta_{mb}(z, k, b) = \frac{\beta(1 - \pi_d)E_z \left[(\lambda'_d + \lambda'_b \phi) \left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1 - \alpha)\eta}} \right]}{\mathbb{E} \left[\left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1 - \alpha)\eta}} \right]}$$

Note that for an unconstrained firm $\Delta_{mc} = 1$ and $\Delta_{mb} = 0$. Note that greater financial frictions reduce Δ_{mc} thus increasing MPK, reflecting borrowing frictions. Alternatively, Δ_{mb} falls, reflecting the increased precautionary savings motive. Recall, that expanding capital can help alleviate the borrowing constraint tomorrow. Define

$$\Delta_k(z, k, b) = \Delta_{mc}(z, k, b) (1 + \Delta_{mb}(z, k, b))$$

Therefore, capital policy can be expressed as

$$k' = \left(\frac{\alpha}{1 - \alpha} W \right)^{1 - \alpha} Y^{\frac{1}{\eta}} \left\{ \frac{\beta}{1 - \beta(1 - \delta)} \mathbb{E} \left[\left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1 - \alpha)\eta}} \right] \right\}^{1 - \alpha} \Delta_k(z, k, b)^{1 - \alpha}$$

A.4.3 Capital Wedge

Define the user-cost $R = \frac{1}{\beta} - 1 + \delta$. Rewriting capital above:

$$k' = \left(\frac{\alpha}{1-\alpha} WY^{\frac{1}{1-\alpha}} \right)^{1-\alpha} \left\{ \frac{1}{R} \mathbb{E} \left[\left(\frac{q(z', k')}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \right] \right\}^{1-\alpha} \Delta_k(z, k, b)^{1-\alpha}$$

Continuing, an expression for the micro-level capital wedge:

$$\begin{aligned} \frac{R(k'_i)^{\frac{1}{1-\alpha}}}{\mathbb{E}[p_i y_i | z]} &= \frac{\frac{\alpha}{1-\alpha} WY^{\frac{1}{1-\alpha}} \mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \right] \Delta_k(z_i, k_i, b_i)}{\mathbb{E} \left[m(q(z'_i, k'_i)) \frac{WY^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(z'_i, k'_i)}{z'_i (k'_i)^\alpha} \right)^{\frac{1}{1-\alpha}} \mid z \right]} \\ \frac{R(k'_i)^{\frac{1}{1-\alpha}}}{\mathbb{E}[p_i y_i | z]} &= \frac{\alpha \mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \right] \Delta_k(z_i, k_i, b_i)}{\mathbb{E} \left[m(q(z'_i, k'_i)) \left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{1-\alpha}} \mid z \right] (k'_i)^{\frac{\alpha}{\alpha-1}}} \\ \frac{Rk'_i}{\mathbb{E}[p_i y_i | z]} &= \frac{\alpha \mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \right] \Delta_k(z_i, k_i, b_i)}{\mathbb{E} \left[m(q(z'_i, k'_i)) \left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{1-\alpha}} \mid z \right]} \end{aligned}$$

Combining with aggregate capital wedge

$$\begin{aligned} \frac{Rk'_i}{\mathbb{E}[p_i y_i | z]} \frac{Y}{RK} &= \mathcal{U} \frac{\mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \mid z \right] \Delta_k(z_i, k_i, b_i)}{\mathbb{E} \left[m(q(z'_i, k'_i)) \left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{1-\alpha}} \mid z \right]} \\ \frac{1}{\mathcal{U}} \frac{k'_i}{K} &= \mathbb{E}[p_i q_i | z] \frac{\mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \mid z \right]}{\mathbb{E} \left[m(q(z'_i, k'_i)) \left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{1-\alpha}} \mid z \right]} \Delta_k(z_i, k_i, b_i) \\ \frac{1}{\mathcal{U}} &= \int \mathbb{E}[p_i q_i | z] \frac{\mathbb{E} \left[\left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{(1-\alpha)\eta}} \mid z \right]}{\mathbb{E} \left[m(q(z'_i, k'_i)) \left(\frac{q(z'_i, k'_i)}{z'_i} \right)^{\frac{1}{1-\alpha}} \mid z \right]} \Delta_k(z_i, k_i, b_i) di \end{aligned}$$

A.4.4 Representative Firm

Labor from the perspective of $k' = k(z, k, b)$

$$\begin{aligned}
\ell(z', z, k, b) &= \left(\frac{Y q(z', k'(z, k, b))}{z' k(z, k, b)^{\alpha\eta}} \right)^{\frac{1}{(1-\alpha)\eta}} \\
&= Y^{\frac{1}{(1-\alpha)\eta}} \left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} k'(z, k, b)^{\frac{\alpha}{\alpha-1}} \\
&= Y^{\frac{1}{\eta}} \left(\frac{\alpha}{1-\alpha} W \right)^{-\alpha} \left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} M_k(z, k, b)^{-\alpha}
\end{aligned}$$

If $\mu(z, k, b)$ is the ergodic distribution

$$\begin{aligned}
L &= Y^{\frac{1}{\eta}} \left(\frac{\alpha}{1-\alpha} W \right)^{-\alpha} \int E_z \left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} M_k(z, k, b)^{-\alpha} d\mu(z, k, b) \\
K &= \left(\frac{\alpha}{1-\alpha} W \right)^{1-\alpha} Y^{\frac{1}{\eta}} \int M_k(z, k, b)^{1-\alpha} d\mu(z, k, b)
\end{aligned}$$

Conjecture the representative firm: $Y = Z (K^\alpha L^{1-\alpha})^\eta$

$$\begin{aligned}
Y &= Z \left[\left(\frac{\alpha}{1-\alpha} W \right)^{(1-\alpha)\alpha} Y^{\frac{\alpha}{\eta}} \left(\int M_k(z, k, b)^{1-\alpha} d\mu(z, k, b) \right)^\alpha \right. \\
&\quad \left. Y^{\frac{1-\alpha}{\eta}} \left(\frac{\alpha}{1-\alpha} W \right)^{-\alpha(1-\alpha)} \left(\int E_z \left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} M_k(z, k, b)^{-\alpha} d\mu(z, k, b) \right)^{1-\alpha} \right]^\eta \\
Z &= \left[\left(\int M_k(z, k, b)^{1-\alpha} d\mu(z, k, b) \right)^\alpha \left(\int E_z \left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} M_k(z, k, b)^{-\alpha} d\mu(z, k, b) \right)^{1-\alpha} \right]^{-\eta}
\end{aligned}$$

Furthermore,

$$\begin{aligned}
Z &= \left[\left(\int \mathbb{E} \left[\left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \middle| z \right]^{1-\alpha} \Delta_k(z, k, b)^{1-\alpha} d\mu(z, k, b) \right)^\alpha \right. \\
&\quad \left. \left(\int \mathbb{E} \left[\left(\frac{q(z', k'(z, k, b))}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \middle| z \right]^{1-\alpha} \Delta_k(z, k, b)^{-\alpha} d\mu(z, k, b) \right)^{1-\alpha} \right]^{-\eta}
\end{aligned}$$

A.4.5 Aggregate Price Relation

Go back to the aggregate markup

$$\begin{aligned}
\frac{1}{\mathcal{M}} &= \int p_i q_i \frac{1}{m_i} di \\
&= \int p_i q_i \frac{MC_i}{p_i} di \\
&= \int q_i MC_i di \\
&= \int \frac{WY^{\frac{1}{(1-\alpha)\eta}-1}}{(1-\alpha)\eta} \left(\frac{q_i}{z_i}\right)^{\frac{1}{(1-\alpha)\eta}} k_i^{\frac{\alpha}{\alpha-1}} di \\
&= \int \frac{WY^{\frac{1}{(1-\alpha)\eta}-1}}{(1-\alpha)\eta} \left(\frac{q_i}{z_i}\right)^{\frac{1}{(1-\alpha)\eta}} (M_i^k)^{-\alpha} \left(W \frac{\alpha}{1-\alpha}\right)^{-\alpha} Y^{-\frac{\alpha}{(1-\alpha)\eta}} di \\
&= \left(W \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{Y^{\frac{1}{\eta}-1}}{\alpha\eta} \int \left(\frac{q_i}{z_i}\right)^{\frac{1}{(1-\alpha)\eta}} (M_i^k)^{-\alpha} di \\
&= \left(W \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{Y^{\frac{1}{\eta}-1}}{\alpha\eta} Z^{\frac{-1}{(1-\alpha)\eta}} \left(\int (M_i^k)^{1-\alpha} di\right)^{\frac{\alpha}{\alpha-1}} \\
&= \left(W \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{Y^{\frac{1}{\eta}-1}}{\alpha\eta} Z^{\frac{-1}{(1-\alpha)\eta}} K^{\frac{\alpha}{\alpha-1}} \left(\frac{\alpha}{1-\alpha} W\right)^{-\alpha} Y^{\frac{1}{\eta} \frac{\alpha}{1-\alpha}} \\
\frac{1}{\mathcal{M}} &= \frac{WY^{\frac{1}{(1-\alpha)\eta}-1}}{(1-\alpha)\eta} Z^{-\frac{1}{(1-\alpha)\eta}} K^{\frac{\alpha}{\alpha-1}}
\end{aligned}$$

A.5 Planner's Problem

A.5.1 Constrained Aggregate K and L

The planner chooses capital allocations $k^p(z)$ subject to capital and labor constraint. Since capital is chosen prior to realization of productivity, they will also chose optimal output $y^p(z', k^p(z))$ for a firm which transitions from productivity z to z' and was allocated $k^p(z)$ initially. Additionally, they choose output for entrants. Define $y^p(z', z) = y^p(z', k^p(z))$. The planners problem is

$$\begin{aligned}
& \max_{y^p(z', z), k^p(z), Y^p} Y^p \\
& \text{s.t.} \quad \pi_d \int \Upsilon \left(\frac{y^p(z, k_0)}{Y^p} \right) dF_0(z) + (1 - \pi_d) \int \mathbb{E} \left[\Upsilon \left(\frac{y^p(z', z)}{Y^p} \right) \middle| z \right] dF_{ss}(z) = 1 \\
& \quad \pi_d \int \left(\frac{y^p(z, k_0)}{z k_0} \right)^{\frac{1}{1-\alpha}} dF_0(z) + (1 - \pi_d) \int \mathbb{E} \left[\left(\frac{y^p(z', z)}{z' k^p(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \middle| z \right] dF_{ss}(z) \leq L \\
& \quad \pi_d k_0 + (1 - \pi_d) \int k^p(z) dF_{ss}(z) \leq K
\end{aligned}$$

They maximize Y^p . I define F_{ss} as the Ergodic distribution of active producers with productivity z . The first constraint is the Kimball technology. The constraint accounts for new producers entering with capital k_0 and productivity drawn from F_0 , as well as surviving producers and their capital allocation $k^p(z)$. The final two constraints are the labor and capital constraints, respectively.

First-order condition for Y :

$$1 = \frac{\lambda_A}{Y} \left[\pi_d \int \Upsilon'(q(z, k_0)) q(z, k_0) dF_{ent}(z) + (1 - \pi_d) \int \mathbb{E} \left[\Upsilon'(q(z', z)) q(z', z) \middle| z \right] dF_{ss}(z) \right]$$

Let $D \equiv \frac{\lambda_A}{Y}$. First-order condition for $q(z, k_0)$ and $q(z', z)$

$$\begin{aligned}
D\Upsilon'(q(z, k_0)) y(z, k_0) &= \frac{\Lambda_L}{1 - \alpha} \left(\frac{y(z, k_0)}{z k_0^\alpha} \right)^{\frac{1}{1-\alpha}} \\
D\Upsilon'(q(z', z)) y(z', z) &= \frac{\Lambda_L}{1 - \alpha} \left(\frac{y(z', z)}{z k^p(z)^\alpha} \right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

Integrating across active producers we get $\Lambda_l = \frac{Y}{L}(1 - \alpha)$. Using same aggregation $Y =$

$$ZK^\alpha L^{1-\alpha}$$

$$\begin{aligned}\frac{Y}{L} \left(\frac{y(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} &= D\Upsilon' \left(\frac{y(z', z)}{Y} \right) y(z', z) \\ \frac{Y^{\frac{1}{1-\alpha}}}{L} \left(\frac{q(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} &= D\Upsilon' (q(z', z)) q(z', z) \\ Z^{\frac{1}{1-\alpha}} K^{\frac{\alpha}{1-\alpha}} \left(\frac{q(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} &= D\Upsilon' (q(z', z)) q(z', z) \\ \left(\frac{q(z', z)}{\frac{z'}{Z} \left(\frac{k(z)}{K} \right)^\alpha} \right)^{\frac{1}{1-\alpha}} &= D\Upsilon' (q(z', z)) q(z', z)\end{aligned}$$

Restating for surviving incumbents and entrants

$$\begin{aligned}D\Upsilon' (q(z, k_0)) q(z, k_0) &= \left(\frac{q(z, k_0)}{\frac{z}{Z} \left(\frac{k_0}{K} \right)^\alpha} \right)^{\frac{1}{1-\alpha}} \\ D\Upsilon' (q(z', z)) q(z', z) &= \left(\frac{q(z', z)}{\frac{z'}{Z} \left(\frac{k(z)}{K} \right)^\alpha} \right)^{\frac{1}{1-\alpha}}\end{aligned}$$

First-order condition for $k(z)$

$$0 = \lambda_L (1 - \pi_d) \mathbb{E} \left[\left(\frac{y(z', z)}{z k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{\alpha}{1 - \alpha} \frac{1}{k(z)} \middle| z \right] - \lambda_K (1 - \pi_d)$$

Re-writing

$$k(z) \lambda_K = \alpha \frac{\Lambda_L}{1 - \alpha} \mathbb{E} \left[\left(\frac{y(z', z)}{z k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \middle| z \right]$$

Plug in FOC for $q(z', z)$

$$\alpha Y D \mathbb{E} [\Upsilon' (q(z', z)) q(z', z)] = \tilde{\Lambda}_K k(z)$$

Integrating using F_{ss} and scaling by $1 - \pi_d$

$$\begin{aligned}
(1 - \pi_d)\alpha Y D \int \mathbb{E} [\Upsilon'(q(z', z))q(z'z)] dF_{ss}(z) &= (1 - \pi_d)\tilde{\Lambda}_K \int k(z) dF_{ss}(z) \\
(1 - \pi_d)\frac{\alpha Y}{\hat{\Lambda}_K} D \int \mathbb{E} [\Upsilon'(q(z', z))q(z'z)] dF_{ss}(z) &= K - \pi_d k_0 \\
\frac{\alpha Y}{\hat{\Lambda}_K} D \left\{ \frac{1}{D} - \pi_d \int \Upsilon'(q(z, k_0))q(z, k_0) dF_0(z) \right\} &= K - \pi_d k_0 \\
\frac{\alpha Y}{K - \pi_d k_0} \left\{ 1 - D\pi_d \int \Upsilon'(q(z, k_0))q(z, k_0) dF_0(z) \right\} &= \hat{\Lambda}_K
\end{aligned}$$

Therefore:

$$\begin{aligned}
k(z) &= \frac{\alpha Y}{\hat{\Lambda}_K} D \mathbb{E} [\Upsilon'(q(z', z))q(z'z)] \\
k(z) &= \frac{K - \pi_d k_0}{1 - D\pi_d \int \Upsilon'(q(z, k_0))q(z, k_0) dF_0(z)} D \mathbb{E} [\Upsilon'(q(z', z))q(z'z)]
\end{aligned}$$

In the text, I define $\alpha^p = \frac{1}{K} \frac{K - \pi_d k_0}{1 - D\pi_d \int \Upsilon'(q(z, k_0))q(z, k_0) dF_0(z)}$ such that capital policy can be expressed

$$k(z) = \alpha^p K D \mathbb{E} [\Upsilon'(q(z', z))q(z'z)]$$

Now to back out aggregate productivity

$$\begin{aligned}
L &= \pi_d \int \left(\frac{y(z, k_0)}{z k_0} \right)^{\frac{1}{1-\alpha}} dF_0(z) + (1 - \pi_d) \int \mathbb{E} \left[\left(\frac{y(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \middle| z \right] dF_{ss}(z) \\
Y^{\frac{1}{(1-\alpha)}} Z^{-\frac{1}{(1-\alpha)}} K^{-\frac{\alpha}{1-\alpha}} &= \pi_d \int \left(\frac{y(z, k_0)}{z k_0} \right)^{\frac{1}{1-\alpha}} dF_0(z) + (1 - \pi_d) \int \mathbb{E} \left[\left(\frac{y(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \middle| z \right] dF_{ss}(z) \\
(Z K^\alpha)^{\frac{-1}{(1-\alpha)}} &= \pi_d \int \left(\frac{q(z, k_0)}{z k_0} \right)^{\frac{1}{1-\alpha}} dF_0(z) + (1 - \pi_d) \int \mathbb{E} \left[\left(\frac{q(z', z)}{z' k(z)^\alpha} \right)^{\frac{1}{1-\alpha}} \middle| z \right] dF_{ss}(z) \\
Z &= \left[\pi_d \int \left(\frac{q(z, k_0)}{z} \right)^{\frac{1}{1-\alpha}} \left(\frac{k_0}{K} \right)^{\frac{\alpha}{\alpha-1}} dF_0(z) + \int \mathbb{E} \left[\left(\frac{q(z', z)}{z'} \right)^{\frac{1}{(1-\alpha)\eta}} \left(\frac{k(z)}{K} \right)^{\frac{\alpha}{\alpha-1}} \middle| z \right] dF_{ss}(z) \right]
\end{aligned}$$

Computing Efficient Allocation. Define $A = \frac{Z^{\frac{1}{(1-\alpha)}} K^{\frac{\alpha}{1-\alpha}}}{D}$. One can characterize solutions for $q(z', z)$ as functions of A and $k(z)$. Given capital allocations $k(z)$, one finds A such the Kimball aggregator equals one. Becomes A and $k(z)$ are determined simultaneously, be-

ginning with a guess for $k(z)$ find A and use A to get q , D , and update $k(z)$ until iterations converge. With $k(z)$ and q solved we can compute D and Z using the equations above.

Incumbent Firms. The distribution of incumbent firms $F_{ss}(z)$ is pinned down by the law of motion:

$$\begin{aligned}\hat{F}_{ss}(z') &= (1 - \pi_d) \int Q_z(z'|z) d\hat{F}(z) + \pi_d \int Q_z(z'|z) dF_0(z) \\ F_{ss}(z) &= (1 - \pi_d) \hat{F}_{ss}(z) + \pi_d F_0(z)\end{aligned}$$

If we discretize the above expression for $\hat{F}_{ss}$ we can get a closed form solution

$$\begin{aligned}\hat{\mathbf{F}} &= (1 - \pi_d) \mathbf{Q} \hat{\mathbf{F}} + \pi_d \mathbf{Q} \mathbf{F}_0 \\ &= [1 - (1 - \pi_d) \mathbf{Q}]^{-1} \pi_d \mathbf{Q} \mathbf{F}_0\end{aligned}$$

A.6 Constrained Efficiency

A.6.1 Lagrangian

The sequence problem for the manager:

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \{ \log C_t - \Psi L_t \} + \beta^t \Lambda_{L,t} \left\{ L_t - Y_t^{\frac{1}{1-\alpha}} \int \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} d\mu_t(x) \right\} \\ &+ \beta^t \Lambda_{R,t} \{ C_t - Y_t + I_t + f_c \} \\ &+ \beta^t \Lambda_{W,t} \{ W_t - \Psi C_t \} \\ &+ \beta^t \Lambda_{K,t} \left\{ \int \Upsilon(q_t(x)) d\mu_t(x) - 1 \right\} + \beta^t \Lambda_{D,t} \left\{ 1 - D_t \int \Upsilon'(q_t(x)) q_t(x) d\mu_t(x) \right\} \\ &+ \int \beta^t \lambda_{d,t}(x) \left\{ \Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'_t(x) + q_{b,t} b'_t(x) - b - f_c \right\} d\mu_t(x) \\ &+ \int \beta^t \lambda_{b,t}(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} \right] - b'(x) \right\} d\mu_t(x)\end{aligned}$$

Where

$$\begin{aligned}I_t &= \int k'_t(x) d\mu_t(x) - (1 - \delta) \int k d\mu_t(x) \\ &= \int k'_t(x) d\mu_t(x) - (1 - \delta) \int k'_{t-1}(x) d\mu_{t-1}(x)\end{aligned}$$

Note for simplicity, I ignore exit. In principle, this will be necessary so that firms don't outlive their financial frictions with certainty on average. It's inclusion is however trivial.

A.6.2 Capital Policy

First-order condition for $k'_t(x)$

$$\begin{aligned}
0 = & -\frac{1}{C_t}\mu_t(x) + \Lambda_{W,t}\mu_t(x)\Psi - \mu_t(x)\lambda_{d,t}(x) \\
& + \beta \left\{ \frac{1}{C_{t+1}}(1-\delta)\mu_t(x) - \Psi\Lambda_{W,t+1}(1-\delta)\mu_t(x) + E \left[\Lambda_{L,t+1} \frac{\alpha}{1-\alpha} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right] \mu_t(x) \right. \\
& + \beta \left\{ E \left[\lambda_{d,t+1}(x') \left(1 - \delta + \frac{\alpha}{1-\alpha} W_{t+1} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right) \right] \mu_t(x) \right\} \\
& \left. + \beta \left\{ E \left[\lambda_{b,t+1}(x') \phi \left(\frac{\alpha}{1-\alpha} W_{t+1} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right) \right] \mu_t(x) \right\} \right\}
\end{aligned}$$

Factor out $C_t/\mu_t(x)$

$$\begin{aligned}
0 = & -1 + \tilde{\Lambda}_{W,t}\Psi - \tilde{\lambda}_{d,t}(x) \\
& + \beta \frac{C_t}{C_{t+1}} \left\{ (1-\delta) - \Psi(1-\delta)\tilde{\Lambda}_{W,t+1} + E \left[\tilde{\Lambda}_{L,t+1} \frac{\alpha}{1-\alpha} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right] \right\} \\
& + \beta \frac{C_t}{C_{t+1}} \left\{ E \left[\tilde{\lambda}_{d,t+1}(x') \left(1 - \delta + \frac{\alpha}{1-\alpha} W_{t+1} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right) \right] \right\} \\
& + \beta \frac{C_t}{C_{t+1}} \left\{ E \left[\tilde{\lambda}_{b,t+1}(x') \phi \left(\frac{\alpha}{1-\alpha} W_{t+1} Y_{t+1}^{\frac{1}{1-\alpha}} \left(\frac{q_{t+1}(x')}{z'(k'_t(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'_t(x)} \right) \right] \right\}
\end{aligned}$$

In steady state

$$\begin{aligned}
0 = & -1 + \tilde{\Lambda}_W\Psi - \tilde{\lambda}_d(x) \\
& + \beta \left\{ (1-\delta) - \Psi(1-\delta)\tilde{\Lambda}_W + E \left[\tilde{\Lambda}_L \frac{\alpha}{1-\alpha} Y^{\frac{1}{1-\alpha}} \left(\frac{q(x')}{z'(k'(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'(x)} \right] \right\} \\
& + \beta \left\{ E \left[\tilde{\lambda}_d(x') \left(1 - \delta + \frac{\alpha}{1-\alpha} W Y^{\frac{1}{1-\alpha}} \left(\frac{q(x')}{z'(k'(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'(x)} \right) \right] \right\} \\
& + \beta \left\{ E \left[\tilde{\lambda}_b(x') \phi \left(\frac{\alpha}{1-\alpha} W Y^{\frac{1}{1-\alpha}} \left(\frac{q(x')}{z'(k'(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'(x)} \right) \right] \right\}
\end{aligned}$$

Note that $\Lambda_L = \Psi$, hence $\tilde{\Lambda}_L = W$ by the household's labor first-order condition

$$0 = -1 + \tilde{\Lambda}_W \Psi(1 - \beta(1 - \delta)) - \tilde{\lambda}_d(x) \\ + \beta \left\{ E \left[\left(1 + \tilde{\lambda}_d(x') \right) \right] (1 - \delta) + E \left[\left(1 + \tilde{\lambda}_d(x') + \tilde{\lambda}_b(x') \phi \right) \left(\frac{\alpha}{1 - \alpha} W Y^{\frac{1}{1-\alpha}} \left(\frac{q(x')}{z' (k'(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'(x)} \right) \right] \right\}$$

First-order condition for W , define $\lambda_f = \lambda_d + \phi \lambda_b$

$$0 = \Lambda_W - \int (\lambda_d(x) + \phi \lambda_b(x)) \ell(x) d\mu(x) \\ \Lambda_W = L E_\ell(\lambda_f)$$

First-order condition of capital becomes

$$0 = -1 + L E_\ell(\tilde{\lambda}_f) \Psi(1 - \beta(1 - \delta)) - \tilde{\lambda}_d(x) \\ + \beta \left\{ E \left[\left(1 + \tilde{\lambda}_d(x') \right) \right] (1 - \delta) + E \left[\left(1 + \tilde{\lambda}_d(x') + \tilde{\lambda}_b(x') \phi \right) \left(\frac{\alpha}{1 - \alpha} W Y^{\frac{1}{1-\alpha}} \left(\frac{q(x')}{z' (k'(x))^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{k'(x)} \right) \right] \right\}$$

rearrange to get Proposition 2.

For completeness, the debt FOC

$$0 = \lambda_{d,t}(x) q_{b,t} \mu_t(x) - \lambda_{b,t}(x) \mu_t(x) - \beta E[\lambda_{d,t+1}(x')] \mu_t(x) \\ \lambda_{d,t}(x) q_{b,t} = \lambda_{b,t}(x) + \beta E[\lambda_{d,t+1}(x')] \\ \tilde{\lambda}_{d,t}(x) q_{b,t} = \tilde{\lambda}_{b,t}(x) + \beta \frac{C_t}{C_{t+1}} E[\tilde{\lambda}_{d,t+1}(x')]$$

In steady state,

$$\tilde{\lambda}_d(x) \beta = \tilde{\lambda}_b(x) + \beta E[\tilde{\lambda}_d(x')]$$

Therefore, debt policy is identical to the competitive equilibrium.

A.6.3 Aggregate Markups

First-order condition for D :

$$\begin{aligned} 0 &= -\Lambda_D \int \Upsilon'(q(s))q(s)d\mu(s) + \int \lambda_f(s)Y\Upsilon'(q(s))q(s)d\mu(s) \\ \frac{\Lambda_D}{Y} &= \frac{\int \lambda_f(s)\Upsilon'(q(s))q(s)d\mu(s)}{\int \Upsilon'(q(s))q(s)d\mu(s)} \\ &= E_s(\lambda_f) \end{aligned}$$

First-order condition for Y

$$\begin{aligned} 0 &= \frac{1}{C_t} - \frac{\Lambda_{L,t}}{1-\alpha} Y_t^{\frac{\alpha}{1-\alpha}} \int \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} d\mu_t(x) - \Lambda_{W,t}\Psi \\ &\quad + \int \lambda_{f,t}(x) \left\{ \Upsilon'(q_t(x))D_t q_t(x) - \frac{1}{1-\alpha} W_t Y_t^{\frac{\alpha}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \right\} d\mu_t(x) \\ 0 &= 1 - \frac{\tilde{\Lambda}_{L,t}}{1-\alpha} Y_t^{\frac{\alpha}{1-\alpha}} \int \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} d\mu_t(x) - \tilde{\Lambda}_{W,t}\Psi + \\ &\quad + \int \tilde{\lambda}_{f,t}(x) \left\{ \Upsilon'(q_t(x))D_t q_t(x) - \frac{1}{1-\alpha} W_t Y_t^{\frac{\alpha}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \right\} d\mu_t(x) \end{aligned}$$

Again, as before study the steady state

$$\begin{aligned} 0 &= 1 - \frac{W}{1-\alpha} Y^{\frac{\alpha}{1-\alpha}} \int \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} d\mu(x) - \tilde{\Lambda}_W \Psi \\ &\quad + \int \tilde{\lambda}_f(x) \left\{ \Upsilon'(q(x))Dq(x) - \frac{1}{1-\alpha} WY^{\frac{\alpha}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \right\} d\mu(x) \\ 0 &= 1 - \frac{W}{1-\alpha} Y^{\frac{\alpha}{1-\alpha}} \int \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} d\mu(x) - \Psi L E_\ell(\tilde{\lambda}_f) + E_s(\tilde{\lambda}_f) - \frac{1}{1-\alpha} \frac{WL}{Y} E_\ell \\ \frac{1}{1-\alpha} \frac{WL}{Y} (1 + E_\ell(\tilde{\lambda}_f)) &= 1 - \Psi L E_\ell(\tilde{\lambda}_f) + E_s(\tilde{\lambda}_f) \\ \frac{1}{1-\alpha} \frac{WL}{Y} &= \frac{1 - \Psi L E_\ell(\tilde{\lambda}_f) + E_s(\tilde{\lambda}_f)}{1 + E_\ell(\tilde{\lambda}_f)} \end{aligned}$$

Therefore, the aggregate markup is

$$\mathcal{M} = \frac{1 + E_\ell(\tilde{\lambda}_f)}{1 - \Psi L E_\ell(\tilde{\lambda}_f) + E_s(\tilde{\lambda}_f)}$$

A.6.4 Markups

The first-order condition for $q(x)$, suppress t since the decision is static:

$$\begin{aligned} 0 &= -\frac{\Psi Y^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} \mu(x) + \Lambda_K \Upsilon'(q(x)) \mu(x) - \Lambda_D D \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} \mu(x) \\ &\quad + \lambda_f(x) \left[DY \Upsilon''(q(x)) q(x) + DY \Upsilon'(q(x)) - \frac{W}{1-\alpha} Y^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} \right] \mu(x) \\ 0 &= -\frac{W Y^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} + C \Lambda_K \Upsilon'(q(x)) - C \frac{\Lambda_D}{Y} DY \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} \\ &\quad + C \lambda_f(x) DY \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} - C \lambda_f(x) \frac{W}{1-\alpha} Y^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} \\ 0 &= -\frac{W Y^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} + C \Lambda_K \Upsilon'(q(x)) - C E_s(\lambda_f(x)) DY \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} \\ &\quad + C \lambda_f(x) DY \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} - C \lambda_f(x) \frac{W}{1-\alpha} Y^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} \end{aligned}$$

In the last step, I replaced Λ_D . Next integrate this express to recover the multiplier on the Kimball aggregate Λ_K

$$\begin{aligned} (1 + \tilde{\lambda}_f(x)) \frac{W Y^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} &= \tilde{\Lambda}_K \Upsilon'(q(x)) q(x) + DY \left(\tilde{\lambda}_f(x) - E_s(\tilde{\lambda}_f) \right) \{ \Upsilon''(q(x)) q(x) + \Upsilon'(q(x)) \} q(x) \\ (1 + \tilde{\lambda}_f(x)) \frac{W Y^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} &= \frac{\tilde{\Lambda}_K}{Y} \Upsilon'(q(x)) q(x) + D \Upsilon'(q(x)) q(x) \left(\tilde{\lambda}_f(x) - E_s(\tilde{\lambda}_f) \right) \left[\frac{\theta(x) - 1}{\theta(x)} \right] \\ \frac{1}{1-\alpha} \frac{WL}{Y} + \frac{1}{1-\alpha} \frac{WL}{Y} E_\ell(\tilde{\lambda}_f) &= \frac{\tilde{\Lambda}_K}{Y} \int \Upsilon'(q(x)) q(x) d\mu(x) + Cov_s \left(\tilde{\lambda}_f, \frac{\theta(x) - 1}{\theta(x)} \right) \\ \frac{1}{1-\alpha} \frac{WL}{Y} (1 + E_\ell(\tilde{\lambda}_f)) &= \frac{\tilde{\Lambda}_K}{Y} \frac{1}{D} + Cov_s \left(\tilde{\lambda}_f, \frac{\theta(x) - 1}{\theta(x)} \right) \\ 1 - \Psi L E_\ell(\tilde{\lambda}_f) + E_s(\tilde{\lambda}_f) &= \frac{\tilde{\Lambda}_K}{Y} \frac{1}{D} + Cov_s \left(\tilde{\lambda}_f, \frac{\theta(x) - 1}{\theta(x)} \right) \\ 1 - \Psi L E_\ell(\tilde{\lambda}_f) + Cov_s(\tilde{\lambda}_f, \theta^{-1}) + E_s(\tilde{\lambda}_f) &= \frac{\tilde{\Lambda}_K}{Y} \frac{1}{D} \end{aligned}$$

Returning to FOC of $q(x)$

$$\begin{aligned}
(1 + \tilde{\lambda}_f(x)) \frac{WY^{\frac{1}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} &= \tilde{\Lambda}^K \Upsilon'(q(x)) + DY \left(\tilde{\lambda}_f(x) - E_s \left(\tilde{\lambda}_f \right) \right) \{ \Upsilon''(q(x))q(x) + \Upsilon'(q(x)) \} \\
(1 + \tilde{\lambda}_f(x)) \frac{WY^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} &= D\Upsilon'(q(s)) \left\{ \frac{\tilde{\Lambda}^K}{Y} \frac{1}{D} + \left(\tilde{\lambda}_f(x) - E_s \left(\tilde{\lambda}_f \right) \right) \left(1 - \frac{1}{\theta(s)} \right) \right\} \\
(1 + \tilde{\lambda}_f(x)) \frac{WY^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1}{q(x)} &= D\Upsilon'(q(s)) \left\{ 1 + \mathbf{\Lambda}_K - \mathbf{\Lambda}_D \frac{\theta(x) - 1}{\theta(x)} + \tilde{\lambda}_f(x) \left(\frac{\theta(x) - 1}{\theta(x)} \right) \right\}
\end{aligned}$$

The markup is therefore

$$\frac{1}{m(x)} = \frac{1}{1 + \tilde{\lambda}_f(x)} \frac{1}{m^{unc}(x)} + \frac{\tilde{\lambda}_f(x)}{1 + \tilde{\lambda}_f(x)} \frac{1}{m^{aut}(x)}$$

where

$$\begin{aligned}
\frac{1}{m^{unc}(x)} &= 1 + \mathbf{\Lambda}_K - \mathbf{\Lambda}_D \frac{\theta(x) - 1}{\theta(x)} \\
\mathbf{\Lambda}_K &= E_s \left(\tilde{\lambda}_d \right) - \Psi L E_\ell \left(\tilde{\lambda}_d \right) - Cov_s \left(\tilde{\lambda}_d, \frac{\theta - 1}{\theta} \right) \\
\mathbf{\Lambda}_D &= E_s \left(\tilde{\lambda}_d \right) \\
\frac{1}{m^{aut}(x)} &= \frac{\theta(x) - 1}{\theta(x)}
\end{aligned}$$

This completes proposition 3.

A.6.5 Value Function

Define $V^P(x) \equiv C \frac{\partial \mathcal{L}}{\partial \mu(x)}$. Begin by taking the partial derivative of the social planner's Lagrangian

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mu_t(x)} = & -\frac{1}{C_t} (k'_t(x) - (1 - \delta)k) - \Lambda_{L,t} Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} + \\ & \Lambda_{W,t} \Psi(k'_t(x) - (1 - \delta)k) + \\ & \Lambda_{K,t} \Upsilon(q_t(x)) - \Lambda_{D,t} D_t \Upsilon'(q_t(x)) q_t(x) + \\ & \lambda_{d,t}(x) \left\{ \Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'_t(x) + q_b b'_t(x) - b - f_c \right\} + \\ & \lambda_{b,t}(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} \right] - b'_t(x) \right\} + \\ & \beta \int \frac{\partial \mu_{t+1}(x')}{\partial \mu_t(x)} \frac{\partial \mathcal{L}}{\partial \mu_{t+1}(x')} dx' \end{aligned}$$

Scale by consumption (or divided by marginal utility of consumption)

$$\begin{aligned} \underbrace{C_t \frac{\partial \mathcal{L}}{\partial \mu_t(x)}}_{V_t^P(x)} = & - (k'_t(x) - (1 - \delta)k) - \tilde{\Lambda}_{L,t} \ell_t(x) + \\ & \tilde{\Lambda}_{W,t} \Psi(k'_t(x) - (1 - \delta)k) + \\ & \tilde{\Lambda}_{K,t} \Upsilon(q_t(x)) - \tilde{\Lambda}_{D,t} D_t \Upsilon'(q_t(x)) q_t(x) + \\ & \tilde{\lambda}_{d,t}(x) \left\{ \Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'_t(x) + q_b b'_t(x) - b - f_c \right\} + \\ & \tilde{\lambda}_{b,t}(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{z k^\alpha} \right)^{\frac{1}{1-\alpha}} \right] - b'_t(x) \right\} + \\ & \beta \frac{C_t}{C_{t+1}} \int \frac{\partial \mu_{t+1}(x')}{\partial \mu_t(x)} \underbrace{C_{t+1} \frac{\partial \mathcal{L}}{\partial \mu_{t+1}(x')}}_{V_{t+1}^P(x)} dx' \end{aligned}$$

Clean up in steady state

$$\begin{aligned}
V^P(x) = & - (k'(x) - (1 - \delta)k) - W\ell(x) + \\
& \tilde{\Lambda}_W \Psi(k'_t(x) - (1 - \delta)k) + \\
& \frac{\Lambda_K C}{DY} DY \Upsilon(q(x)) - C \frac{\Lambda_D}{Y} DY \Upsilon'(q(x)) q(x) + \\
& \tilde{\lambda}_d(x) \left\{ \Upsilon'(q(x)) DY q(x) - WY^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c \right\} + \\
& \tilde{\lambda}_b(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \right] - b'(x) \right\} + \\
& \beta E [V^P(x')|z] \\
V^P(x) = & - (k'(x) - (1 - \delta)k) - W\ell(x) + \\
& (k'(x) - (1 - \delta)k) \Psi \tilde{\Lambda}_W - \mathbf{\Lambda}_D DY \Upsilon'(q(x)) q(x) + \\
& (1 + \mathbf{\Lambda}_K) DY \Upsilon(q(x)) + \\
& \tilde{\lambda}_d(x) \left\{ \Upsilon'(q(x)) DY q(x) - WY^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c \right\} + \\
& \tilde{\lambda}_b(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} \right] - b'(x) \right\} + \beta E [V^P(x')|z]
\end{aligned}$$

To make useful, add and subtract profits and transfers

$$\begin{aligned}
V^P(x) &= DY\Upsilon'(q)q - W\ell(x) + (1 - \delta)k - b - f_c - k'(x) + q_b b'(x) + T^P(x) \\
&\quad - DY\Upsilon'(q)q - T^P(x) \left(1 + \tilde{\lambda}_f(x)\right) + \\
&\quad (k'(x) - (1 - \delta)k) \Psi \tilde{\Lambda}_W - \mathbf{\Lambda}_D DY\Upsilon'(q(x))q(x) + \\
&\quad (1 + \mathbf{\Lambda}_K) DY\Upsilon(q(x)) + \\
&\quad \tilde{\lambda}_d(x) \left\{ \Upsilon'(q(x)) DYq(x) - WY^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c + T^P(x) \right\} - \\
&\quad \tilde{\lambda}_b(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + T^P(x) \right] - b'(x) \right\} + \beta E [V^P(x')|z] \\
V^P(x) &= DY\Upsilon'(q)q - W\ell(x) + (1 - \delta)k - b - f_c - k'(x) + q_b b'(x) + T^P(x) \\
&\quad - T^P(x) \left(1 + \tilde{\lambda}_f(x)\right) - (1 + \mathbf{\Lambda}_D) DY\Upsilon'(q(x))q(x) + (1 + \mathbf{\Lambda}_K) DY\Upsilon(q(x)) + \\
&\quad (k'(x) - (1 - \delta)k) \Psi \tilde{\Lambda}_W + \\
&\quad \tilde{\lambda}_d(x) \left\{ \Upsilon'(q(x)) DYq(x) - WY^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c + T^P(x) \right\} - \\
&\quad \tilde{\lambda}_b(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + T^P(x) \right] - b'(x) \right\} + \beta E [V^P(x')|z]
\end{aligned}$$

Note that without the terms in the second and third line, we can write this as a Bellman equation with the usual constraints akin to the firm's problem. To do this we find the $T^P(x)$ to eliminate these terms:

$$\begin{aligned}
T^P(x) &= \frac{1}{1 + \tilde{\lambda}_d(x)} DY \{ \xi_0 \Upsilon(q(x)) - \xi_1 \Upsilon'(q(x))q(x) \} + \frac{\Psi \tilde{\Lambda}_W}{1 + \tilde{\lambda}_d(x)} (k'(x) - (1 - \delta)k) \\
&\quad \xi_0 = 1 + \mathbf{\Lambda}_K \\
&\quad \xi_1 = 1 + \mathbf{\Lambda}_D
\end{aligned}$$

As a consequence,

$$\begin{aligned}
V^P(x) &= DY\Upsilon'(q)q - W\ell(x) + (1 - \delta)k - b - f_c - k'(x) + q_b b'(x) + T^P(x) \\
&\quad \tilde{\lambda}_d(x) \left\{ \Upsilon'(q(x)) DYq(x) - WY^{\frac{1}{1-\alpha}} \left(\frac{q(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + k(1 - \delta) - k'(x) + q_b b'(x) - b - f_c + T^P(x) \right\} - \\
&\quad \tilde{\lambda}_b(x) \left\{ \phi \left[\Upsilon'(q_t(x)) D_t Y_t q_t(x) - W_t Y_t^{\frac{1}{1-\alpha}} \left(\frac{q_t(x)}{zk^\alpha} \right)^{\frac{1}{1-\alpha}} + T^P(x) \right] - b'(x) \right\} + \beta E [V^P(x')|z]
\end{aligned}$$

Therefore, by inspection if you have the correct social cost in the transfers λ_f , then the firm solving the Bellman equation in proposition 4 will make choices that align with the constrained planner.

B Quantitative

For calibration moments, I use Compustat quarterly panel. I exclude firms in finance, insurance, real estate, and utilities. I construct capital using perpetual inventory method as in Ottonello et al. (2022). I drop annual sales growth above and below 100 percent. I drop observations below 1st and above the 99 percentiles for sales, annual percentage change in capital, and leverage. I keep only observations for which sales, leverage, and capital stock are observed for spells of 10 years or more.