Scope Economies and Allocative Efficiency*

Mahdi Ebsim†

November 5, 2026

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Abstract

Size-dependent distortions misallocate resources, yet in standard models they account for only a small share of cross-country income gaps. These models abstract from firm scope and organization. When firms share management across products, they benefit from scope economies, and revenue per product rises with the number of products. Using product-level data, I find this relationship in the US but not in India, suggesting that firms in the two countries organize production differently. I rationalize this pattern in a model where firms choose their scope and whether to adopt costly non-rival management. Adoption raises productivity, and more so for large firms. I introduce distortions to match the Indian firm-size distribution and show that they deter adoption, rationalizing the absence of scope economies in India. The effects of these distortions are amplified by large firms forgoing non-rival management. With scope economies, distortions can account for 80% of the gap in GDP per worker between the US and India, compared with 20% without.

*I am deeply grateful to my advisors, Virgiliu Midrigan, Diego Perez, Corina Boar, and Mark Gertler, for their guidance, encouragement, and patience. I would like to thank Serdar Birinci, Jaroslav Borovička, Francisco Buera, Bill Dupor, Miguel Faria-e-Castro, Lukas Freund, Simon Gilchrist, Jeremy Greenwood, Aakash Kalyani, Seula Kim, Julian Kozlowski, Yueran Ma, Michael Peters, Thomas Philippon, B. Ravikumar, Hannah Rubinton, Juan M. Sánchez, and Venky Venkateswaran for helpful comments and discussions. This paper contains my own analyses calculated (or derived) based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business. The conclusions drawn from the NielsenIQ data are those of the author and do not reflect the views of NielsenIQ. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

†New York University. Email: ame384@nyu.edu. Website: https://sites.google.com/view/mahdiebsim/

1. Introduction

Income per capita differs greatly across countries. A commonly held view is that firm-level distortions, such as financial frictions, impediments to competition, or government policy, explain these differences. Since these distortions fall more heavily on large firms, they misallocate resources and lower output (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). Although such distortions are pervasive in developing economies, standard models predict that they account for only a small share of income gaps (Hopenhayn, 2014). These models focus on how distortions affect firms' input decisions, taking as given a firm's scope (i.e., the number of products) and organization (i.e., how management is shared within the firm). In practice, scope and organization differ across firms, both within and across countries. Distortions that fall on large firms may also narrow their scope and distort their organization choice.

In this paper, I reassess the implications of size-dependent distortions for cross-country income gaps, taking into account how firms differ in scope and organization. I argue that scope economies amplify the effects of these distortions, which change not only firms' input decisions but also their organization choice. I combine evidence on scope economies in the US and India with a heterogeneous-firm model in which firms choose their scope and whether to adopt non-rival management. Adopters share management across products, so expanding scope reduces the marginal cost of existing products and makes these firms more productive. Because adopters tend to be larger, distortions fall disproportionately on them. When their scope narrows enough, they abandon non-rival management, and the costs of their remaining products rise. I discipline the model with the evidence on scope economies, calibrating it to the US and holding its technology parameters fixed when I introduce distortions to match India. These distortions rationalize the absence of scope economies in India. Eliminating them raises aggregate productivity by about 100%, four times the gain in a recalibrated model without scope economies. With scope economies, distortions can account for 80% of the gap in GDP per worker between the US and India, compared with 20% without them.

Using product-level data, I document that revenue per product rises with the number of products in the US but not in India. This pattern is consistent with scope economies in the US that are absent in India. Two pieces of evidence point to non-rival management, enabled by inputs such as organization capital, as the source of scope economies in the US. First, US firms that use more organization capital relative to physical capital have broader scope. Second, I provide direct evidence from US firms' adoption of firm-wide management software, which centralizes inventory, human resources, and distribution across a firm's product lines. Firms often disclose when this software goes live in their SEC filings. Around these dates, revenue per product rises more steeply with the number of products, with no pre-trend, consistent with the software generating scope economies.

In the model, a continuum of multi-product firms produces differentiated goods subject to CES demand. Firms differ in the productivity of their core product and in the cost of maintaining their scope. They choose how many products to offer and whether to adopt non-

rival management. Each product uses labor, materials, and a managerial input. If a firm does not adopt, the managerial input is labor and must be replicated across products. If a firm does adopt, the managerial input combines labor with organization capital and is shared across products, following Crouzet et al. (2026).

The firm's problem reduces to that of a single-product firm whose productivity is an aggregate over everything it sells. This firm-level productivity increases with scope, and it rises faster if the firm adopts non-rival management, at a rate that depends on the strength of non-rivalry. These productivity gains reflect cost savings, as sharing management across more products lowers the marginal cost of each. Non-rival management therefore gives adopters greater returns to scale. Since adopters are the largest firms, the right tails of the scope and firm-size distributions thicken by an amount that depends on the strength of non-rivalry. By contrast, if adoption raised a firm's productivity by a common proportion, it would shift the tails without thickening them.

Size-dependent distortions affect scope economies along two margins. Along the extensive margin, they push firms below the adoption threshold, forcing them to forgo the productivity gain associated with non-rival management. Along the intensive margin, because adopters have greater returns to scale, distortions fall more heavily on them, reducing the productivity gain from cost savings. In standard models, distortions that leave the aggregate wedge unchanged lower productivity only at second order, through misallocation (Hopenhayn, 2014; Baqaee and Farhi, 2020). In this model, these two margins lower aggregate productivity at first order, by changing how firms organize production.

To discipline the model, I use the firm-size and scope distributions and the relationship between revenue per product and the number of products. In the US, this relationship cannot be read directly as evidence of scope economies, because productive firms both sell more products and earn higher revenue per product. Thus, the cross-sectional elasticity overstates scope economies. In India, this relationship is flat, so the upward bias only reinforces the absence of scope economies.

To correct this upward bias in the US, I exploit the fact that firms sell products across multiple sectors. A large increase in input costs in one sector leads a firm to drop products there. If management is non-rival, the costs of surviving products in other sectors rise, and their revenues fall. Without scope economies, they are unaffected. I use firms' exposure to sectoral input-cost shocks, constructed from input-output tables, as an instrument for scope.

The IV estimate of the elasticity of revenue per product with respect to the number of products is 0.20, about 75% lower than the OLS estimate. In the model, this elasticity is approximately the share of firms that adopt non-rival management times the scope economies of adopters. The firm-size distribution separates the two. Adoption creates a gap in the firm-size distribution at the adoption threshold, and the size of this gap depends on the strength of non-rivalry, which pins down scope economies. Given non-rivalry, the IV estimate implies an adoption rate, and hence the cost of adoption. I show that the scope economies of adopters and

the adoption rate of non-rival management govern the amplification of distortions. Hence, the estimated elasticity maps directly into amplification.

For India, two parameters govern distortions: the degree of size dependence and a common distortion. Size dependence is disciplined by the tail of India's firm-size distribution. Because the strength of non-rivalry is held constant across countries, the common distortion, which reduces adoption, is disciplined by the absence of scope economies in India. I find that scope economies change the inferred distortions. Matching India requires weaker size dependence than in a model without scope economies. Size dependence can be weaker because having fewer adopters already thins the tail. The common distortion must be large enough to deter adoption and eliminate scope economies. Unlike in standard models, a common distortion acts like a size-dependent one, since only large firms adopt. Distortions can rationalize the absence of scope economies in India only if the materials share exceeds one half, so that lower aggregate productivity further discourages adoption.

Calibrating a model without scope economies to the same moments, I find that the effects of size-dependent distortions are muted. With scope economies, holding the adoption rate fixed cuts the productivity gain from eliminating distortions in half. Moreover, if India's adoption rate for non-rival management were identical to the US rate, the effects of distortions would be quantitatively similar to those in a model without scope economies. Therefore, the differential in scope economies is the key source of amplification.

1.1. Related Literature

This work builds on Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) on misallocation. I focus on size-dependent distortions. These stand in for, among other things, government policies that penalize large firms, as studied by Guner et al. (2008) and García-Santana and Pijoan-Mas (2014), and as documented empirically by Martin et al. (2017). They also stand in for financial frictions that misallocate capital (Buera and Shin, 2013; Bau and Matray, 2023). Consistent with the form of size-dependent distortions I model, David and Venkateswaran (2019) find that a large share of capital misallocation stems from a component correlated with productivity. Relative to this work, I study the implications of size-dependent distortions in the context of scope economies. In doing so, I address a concern, raised by Hopenhayn (2014), that large effects from such distortions are difficult to generate without rank reversals. In the presence of scope economies, these distortions can generate large effects while preserving the ranking of firms.

Since scope economies in this model act as productivity gains, this paper also relates to the literature on technology adoption. Bento and Restuccia (2017) show that output losses from distortions at India's level are larger when distortions also deter investment in productivity (53% compared with 37%). Here, the payoff to the investment rises with the number of products a firm sells, which makes the gain measurable in product-level data. Buera et al. (2026) study technology adoption subject to size-dependent distortions, emphasizing the role

of complementarity across firms for amplification. Relative to their work, amplification here arises because the gain from adoption rises with firm scope, so distortions fall on the firms with the most to gain from adoption. Following Bloom et al. (2013), who show that adopting management practices raises firms' productivity, I model adoption as an organization choice rather than as a move toward a technology frontier (Parente and Prescott, 1994; Caselli and Coleman, 2006; Comin and Mestieri, 2018).

This paper is motivated by the literature on management practices and development (Akcigit et al., 2021; Hjort et al., 2026), which I build on by modeling good management practices as a source of scope economies. To generate them, I assume that organization capital allows management to be non-rival within the firm, following work on intangibles (Crouzet et al., 2026; Eisfeldt and Papanikolaou, 2013; Jiang and Rubinton, 2026). Recent work shows that how firms organize production determines their productivity and scale over the course of development (Engbom et al., 2025) and shapes how they adapt to new technologies (Crouzet et al., 2025). I contribute by mapping the gains from firm organization to the elasticity of revenue per product with respect to the number of products. Boehm et al. (2022) show that shared input capabilities shape which industries firms enter, whereas I focus on a shared managerial input whose returns rise with scope. Argente et al. (2025) show that firms with more scalable expertise choose broader scope and are more responsive to demand. My model delivers similar predictions for scope through the adoption of non-rival management, and I derive the implications of these predictions for aggregate productivity across countries.

This paper is also related to the literature on multi-product firms. Bernard et al. (2010) and Goldberg et al. (2010b) document that multi-product firms account for the bulk of output in the US and India, respectively. Recent work shows that product churn underlies firm growth (Argente et al., 2024; Jo and Kim, 2026). I document that while multi-product firms are more common in India than in the US, sales in the US are considerably more concentrated among firms with broad scope. I model multi-product firms similarly to Arkolakis et al. (2021), extending their model to allow for scope economies via non-rival management. Fattal Jaef (2018) studies the implications of multi-product firms for misallocation. In comparison, I show that scope economies are critical for generating substantial amplification.

Lastly, this paper contributes to the estimation of scope economies. Ding (2024) uses export demand shocks to show that knowledge inputs such as R&D and software generate scope economies through their scalability and partial non-rivalry. Koh and Raval (2025) use a production-function approach to show that shared inputs are a source of scope economies, and Khmelnitskaya et al. (2026) identify scope economies using demand-side data. I contribute an identification strategy that uses input-cost shocks and product-level data to isolate exogenous variation in firm scope. Through the model, the resulting estimate disciplines the non-rivalry of management, which governs the degree of amplification.

1.2. Outline

Section 2 documents evidence of scope economies in the US but not in India. Section 3 presents the model, and Section 4 characterizes firms' scope and adoption decisions and their implications for the firm-size distribution. Section 5 develops the IV strategy for identifying scope economies and calibrates the model to the US. Section 6 introduces size-dependent distortions and shows how scope economies amplify their effects. Section 7 quantifies the effects of distortions in India.

2. Data and Motivating Evidence

In this section, I use product-level data from the US and India to motivate studying scope economies. First, the firm-size distribution is thinner-tailed in India than in the US, a pattern commonly read as evidence of size-dependent distortions, and the distribution of firm scope shows the same pattern. Second, revenue per product rises with the number of products in the US but not in India, consistent with scope economies that are present in the US but absent in India. Third, using US data, I provide evidence that these scope economies arise from non-rival managerial inputs. Lastly, I document how size-dependent distortions affect the relationship between revenue per product and the number of products in India.

2.1. Data

This subsection describes the main data sources and the sample construction behind the motivating evidence. Other data sources are described where they are used. Appendix A.1 provides additional details.

US Product Data. US product-level data come from the NielsenIQ Retail Measurement Service (RMS), which records point-of-sale data for products identified by Universal Product Codes (UPCs). These products are consumer packaged goods (CPG), a sector that accounted for approximately 10% of GDP in 2017.[1] RMS records CPG sales in participating grocery, drug, mass merchandiser, and convenience stores rather than all CPG sales; see Argente et al. (2024) for details on its coverage. I aggregate observations to the UPC-year level, identify firms by the owner of the UPC, and construct a firm×UPC×year panel of sales, quantities, and prices from 2006 to 2019.

Indian Product Data. Indian product data come from the Prowess database, compiled by the Centre for Monitoring Indian Economy (CMIE). It covers most publicly traded firms and a large number of private firms, which together represent more than 70% of the organized industrial sector in India (Bau and Matray, 2023). See Goldberg et al. (2010b,a) for early uses of these data to study multi-product firms. In Prowess, firm sales are disaggregated by product, with information on revenues, prices, and quantities, and product-level reporting is extensive.

[1] Based on calculations in PricewaterhouseCoopers LLP (2019).

Goldberg et al. (2010b) report that product-level data are available for 85% of manufacturing firms in Prowess, which account for more than 90% of output, and that product-level sales sum to 92% of total reported output. I construct a firm×product×year panel from 1998 to 2019.

Product Definition. Comparing results across the two datasets requires a consistent definition of a product. UPCs in NielsenIQ RMS are far more granular than CMIE product categories, since they distinguish items by packaging, flavor, and brand. For example, CMIE's category "Instant coffee" does not distinguish among types of instant coffee. I therefore map each NielsenIQ product module, a grouping of UPCs, to a CMIE product category. The resulting crosswalk maps 1,134 NielsenIQ product modules to 318 CMIE product categories in the CPG sector, with a median of one module per category. Whenever I compare the US and India directly, products are defined at the CMIE category level, and $n_{f,t}$ denotes the number of products sold by firm f in year t.

Comparability. The two datasets differ in sectoral coverage, firm definitions, and the share of each firm's sales they capture. First, NielsenIQ data cover only the CPG sector. Prowess data span all manufacturing sectors in India, with about 20% of products and sales in the CPG sector. Second, the datasets differ in which firms they include and in how they identify them. Prowess identifies a firm as a legal entity reporting standalone financial statements and is weighted toward large formal firms, some of which belong to business groups. NielsenIQ includes any legal entity with an issued UPC whose products are sold in participating stores. This does not preclude these entities from being subsidiaries of larger companies. Therefore, the datasets may differ in where they draw firm boundaries. Finally, NielsenIQ data capture only part of each firm's sales, so they can be used only to compare within-country compositions and normalized measures, not levels. I address the first two margins with robustness checks in Section 2.3, and the third by design, since all estimates are computed separately for each country.

Summary Statistics. Table 1 presents summary statistics for both countries.[2] The share of multi-product firms is higher in India (53% of firms, compared with 38% in the US). However, this masks differences in how sales are distributed across firms. Multi-product firms account for 92% of sales in the US but 83% in India, and the top 1% of firms by number of products account for 19% of US sales, about twice their 9% share in India. Sales are therefore more concentrated in high-scope firms in the US than in India. The multi-product shares are close to existing benchmarks. Bernard et al. (2010) find that multi-product firms are 39% of US manufacturing firms and account for 87% of output, and Goldberg et al. (2010b) find that they

[2]In the main text, US products are defined as CMIE categories. The second column of Appendix Table A2 reports summary statistics for the full NielsenIQ sample with products defined as UPCs.

are 47% of Indian manufacturing firms in Prowess and account for 80% of output.[3]

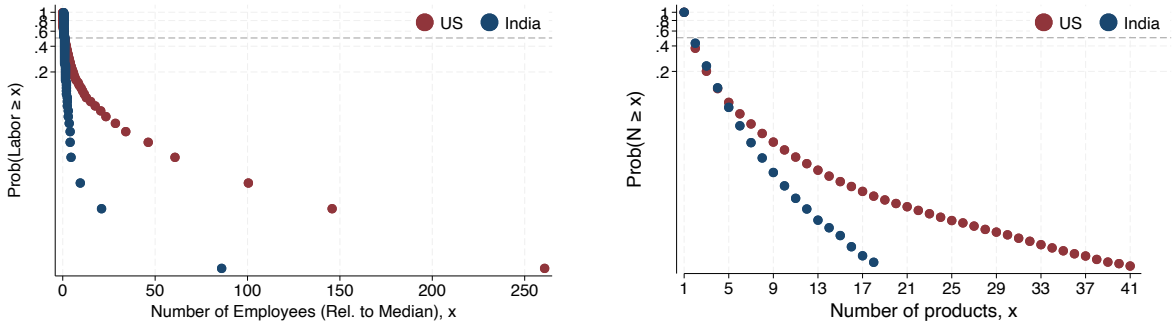
Table 1: Summary Statistics: United States and India

	United States	India
Panel A: Firms		
Share of firms with $n_{f,t} > 1$ (%)	38	53
Sales share of firms with $n_{f,t} > 1$ (%)	92	83
Sales share of firms above 99th pct. of $n_{f,t}$ (%)	19	9
Panel B: Products		
CPG products (% of products)	—	20
CPG products (% of sales)	—	20
Panel C: Sample		
Years	2006–2019	1998–2019
Firms	63,953	10,882
Firm-year observations	456,530	79,525

Notes: Products are CMIE categories (see Section 2.1). All US products are CPG by construction. Shares in Panels A and B are computed for each year; the table reports the mean across years. The 99th percentile of $n_{f,t}$ is computed over the pooled firm-year sample.

2.2. Firm Size and Scope Distributions

Figure 1: Distribution of Firm Scope and Size: United States and India



(a) Firm size

(b) Firm scope

Notes: Size is employment (US: 2007 Compustat, non-financial firms; India: 2005 Economic Census). Scope is the number of products a firm sells in 2007, defined as CMIE categories (see Section 2.1). Panel (b) is truncated at $\Pr(N > x) = 0.001$, where N is the number of products.

Figure 1a plots the complementary CDF of firm employment for India and the US, normalized by the median to make the tails of the two distributions comparable.[4] Relative to the median, large firms are rarer in India than in the US. That is, the Indian distribution has a

[3]Both Bernard et al. (2010) and Goldberg et al. (2010b) use data covering all manufacturing sectors, in the US and India respectively, whereas the NielsenIQ data cover only CPG.

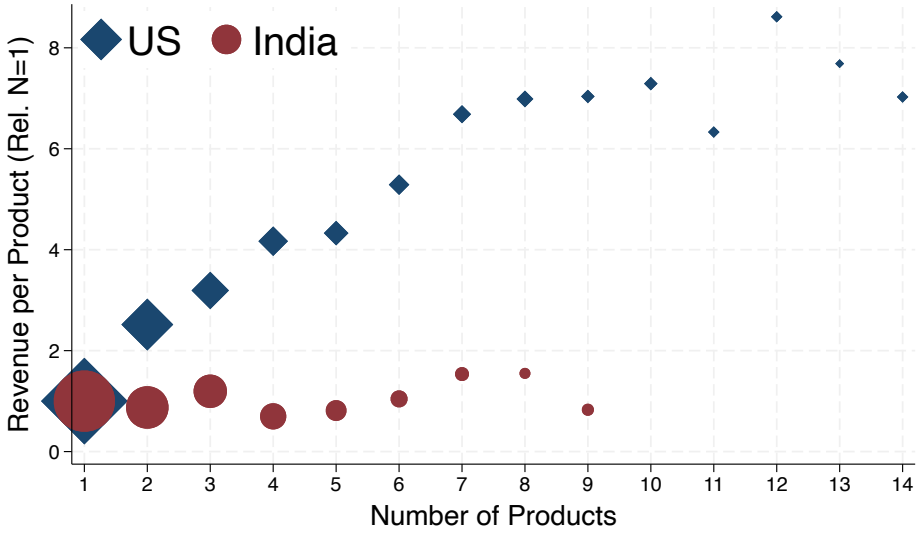
[4]The Indian Economic Census enumerates establishments rather than firms. Compustat covers only publicly listed firms and therefore underrepresents small firms. Appendix A.3 reports alternative measures that address these concerns.

thinner right tail. The patterns in Figure 1a are similar to the documented differences in the firm-size distribution between developed and developing economies (Chen, 2026). These differences are often attributed to size-dependent distortions that fall more heavily on large firms in India. Figure 1b plots the complementary CDF of the number of products. It shows that these patterns are also present in the scope distribution. Hence, if the thin tail of the size distribution reflects distortions, the same distortions may also limit firm scope.

2.3. Revenue per Product and Scope

A firm benefits from scope economies if producing an additional product raises its total costs by less than the cost of producing that product on its own. For example, a firm that shares inputs across products lowers the cost of each product as it adds products. Under scope economies, revenue per product should therefore increase with the number of products a firm sells. Absent scope economies, the relationship should be flat or declining. Figure 2 examines this implication. For each firm-year, I compute revenue per product, $\bar{r}_{f,t} = \frac{1}{n_{f,t}} \sum_{i \in \mathcal{N}_{f,t}} p_{i,f,t} y_{i,f,t}$, where $\mathcal{N}_{f,t}$ is the set of products sold by firm f in year t, and plot its mean at each value of $n_{f,t}$, normalized by the mean for single-product firms in the same country. In the US, revenue per product rises with the number of products a firm sells, whereas in India it is flat. This is consistent with stronger scope economies in the US.

Figure 2: Firm Scale and Scope: United States and India



Notes: The number of products is top-coded at the 99th percentile (14 in the US, 9 in India). The size of each point corresponds to the share of firms with that many products. Products are CMIE categories (see Section 2.1).

However, more productive firms, or firms facing higher demand, may both add products and have higher revenue per product. In Section 5.2, I address this concern by introducing an instrument to isolate changes in firm scope independent of firm-level demand and productivity.

Robustness: Specification and Measurement. The result is robust to a range of alternative specifications and measures. First, it is not driven by differences in industry composition. With year and industry fixed effects, the elasticity of revenue per product with respect to the number of products is positive for the US and close to zero for India (Appendix A.6). The US elasticity remains positive with firm fixed effects, so it is not driven by permanent differences across firms. Second, the result holds for alternative measures of scope, namely UPCs in the US data (Appendix A.7) and the inverse within-firm Herfindahl–Hirschman index following Jovanovic (1993) (Appendix A.5). The remaining exercises are summarized in Appendix A.4.

Robustness: Comparability. The result is also robust along the comparability margins discussed in Section 2.1. First, restricting the Indian data to CPG products does not alter the gap in Figure 2 (Figure A9b). Second, the US relationship remains positive among firms matched to Compustat (Appendix A.7), which are large listed firms and hence closer to the Prowess population, and the Indian relationship remains flat in the Annual Survey of Industries (ASI), a mandatory survey of establishments with broader coverage of Indian manufacturing (Appendix A.8). Finally, defining firms as parent companies, using Compustat parents in the US and business groups in India, leaves the results unchanged.[5]

2.4. Sources of Scope Economies

How multi-product firms organize production matters, because good management practices are potentially non-rival within the firm, allowing the costs of management to be shared across products. Sadun et al. (2025) show that the management scores of plants acquired by high-scoring firms rise after acquisition, suggesting that good practices span the units of a firm.

I characterize investment in management as organization capital (Eisfeldt and Papanikolaou, 2013; Peters and Taylor, 2017), of which software is one component. Software and other organizational inputs are non-rival within the firm (Jiang and Rubinton, 2026), generating scope economies across industries that share them (Ding, 2024). Artificial intelligence allows firms to codify knowledge into software and build organization capital (Eisfeldt et al., 2026; Babina et al., 2026). Once acquired, these inputs can be used across a firm's activities without replication.

Using US data, I link scope economies to non-rival managerial inputs through organization capital and through the adoption of a specific non-rival technology.

Organization Capital. I use the combined NielsenIQ–Compustat panel.[6] Non-rival managerial expenditures, such as those on training, codification, and software, are largely recorded

[5] I do not use the ASI for the main analysis because it is establishment-level data, does not identify owners of multiple establishments, and is not a panel. I do not use the parent-company definitions in the main analysis because Prowess applies a firm's current business group to earlier years, and Compustat omits large private firms in NielsenIQ.

[6] The matched panel contains 629 firms, which account for 35% of NielsenIQ sales and 17% of UPCs. Appendix A.1 describes how NielsenIQ firms are matched to Compustat, and Table A2 reports summary statistics.

in selling, general, and administrative expenses (SG&A). Following Peters and Taylor (2017), I construct the stock of organization capital, $K^{\text{org}}_{f,t}$, from SG&A.[7] For comparison, I also record $K^{\text{ppe}}_{f,t}$, the physical capital (property, plant, and equipment) that firms own. Holding total assets constant, a firm that relies more intensively on organization capital relative to physical capital plausibly generates greater scope economies, and hence has a stronger motive to expand its number of products. I test this by estimating the following specification:

$$\log n_{f,t} = \beta_{\text{org}} \log K^{\text{org}}_{f,t} + \beta_{\text{ppe}} \log K^{\text{ppe}}_{f,t} + \Gamma \log \text{Assets}_{f,t} + \alpha_t + \varepsilon_{f,t},$$

which controls for total assets on the firm's balance sheet. Table 2 shows that β_{org} is positive and β_{ppe} is negative, both significant at the 1% level. This is consistent with a complementarity between non-rival inputs and firm scope.

Table 2: Firm Scope and Capital Composition

	PPE Capital	Org. Capital
Coefficient	-0.372***	0.581***
	(0.115)	(0.147)
Observations	4416	

Notes: Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

ERP Adoption. Next, I focus on Enterprise Resource Planning (ERP) systems, a specific technology with clearly observed adoption dates. ERP systems are integrated software packages that run a firm's main operations, typically including inventory management, human resources, and distribution, from one database. They allow firms to centralize these activities, and the associated administrative roles, across products. The cost of an ERP system does not rise as the firm expands its scope.[8] Because ERP adoption changes firms' internal controls over financial reporting, firms often disclose new ERP systems in their SEC filings, frequently with their "go-live" date.[9] For firms in the combined NielsenIQ–Compustat panel, I parse their filings and extract go-live dates, as explained in Appendix A.2.[10]

To test whether ERP adoption is a source of scope economies, I examine whether the slope of revenue per product with respect to the number of products increases around adoption dates.

[7] I define organization capital investment as 30% of SG&A net of research and development (R&D) expenses, as in Peters and Taylor (2017). Unlike Peters and Taylor (2017), I additionally net out advertising expenses to further isolate the non-rivalry margin. I construct $K^{\text{org}}_{f,t}$ using the perpetual inventory method.

[8] Spending on ERP enters organization capital only in part. Purchased or developed software is capitalized as an intangible asset, while implementation costs such as training and consulting are expensed through SG&A.

[9] Under the Sarbanes–Oxley Act and Item 308(c) of Regulation S-K, firms must disclose material changes in internal control over financial reporting, in Item 9A of the 10-K and Item 4 of the 10-Q. Because only material changes must be disclosed, the sample may be tilted toward larger implementations.

[10] Between 2006 and 2019, I find 86 first adoption events and 420 firms with no adoption event, which serve as controls.

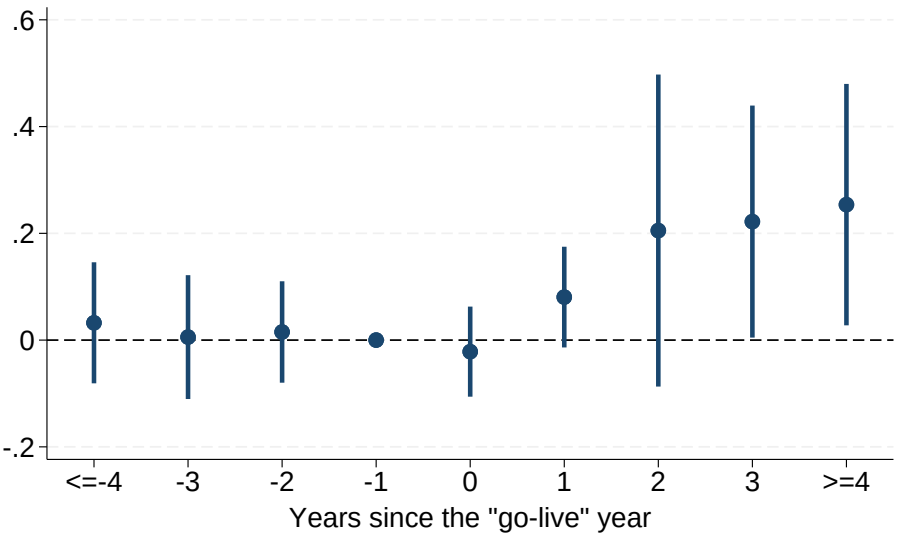
I estimate the following event-study specification:

$$\log p_{i,f,t} y_{i,f,t} = \sum_{j \neq -1} \mathbb{I}(t - \tau_f = j) \Big[\gamma_j + \beta_j \log n_{f,\tau_f - 1} \Big] + \alpha_i + \alpha_t + \varepsilon_{i,f,t}.$$

Here, τ_f is the year of firm f's first observed ERP adoption, so $\mathbb{I}(t - \tau_f = j)$ indicates that year t is j years from adoption, and $n_{f,\tau_f - 1}$ is the scope of the firm in the year preceding adoption. I regress product-level log revenues on these event-time indicators and their interaction with $\log n_{f,\tau_f - 1}$, with product and year fixed effects α_i and α_t, weighting observations by their within-firm sales share. I estimate this event study at the product level, defining products at the UPC level, so as to isolate products that neither exit nor are newly introduced within a narrow window around the adoption date.[11]

Figure 3 plots $\hat{\beta}_j$, the coefficients on scope, for a four-year window around adoption. Slopes are unchanged before adoption but increase significantly afterward. ERP systems are only one of many technologies firms can adopt to exploit scope economies. Nonetheless, this evidence suggests that scope economies arise partly from firms' decisions to adopt technologies that affect their organization of production.

Figure 3: Effect of ERP Adoption × Firm Scope on Revenue per Product



Notes: Estimates of β_j with 95% CI from the event-study specification over a four-year window around the first ERP go-live date; $j = -1$ is the omitted period. Standard errors are clustered at the firm level.

Discussion. This evidence on the sources of scope economies is derived for the US only. Management practices, however, differ across countries. Bloom and Van Reenen (2010) document poorer management practices in developing economies. Since management is partly non-rival

[11] Results are robust to defining products as CMIE categories, as in the results above. See Figure A1.

within the firm, scope economies may differ with the level of development. Firms in developing economies may not adopt modern management practices that generate scope economies because these practices are expensive to employ (Hjort et al., 2026).[12]

2.5. Size-Dependent Distortions and Scope Economies

I provide evidence that size-dependent distortions in India lower the slope of revenue per product with respect to the number of products. Beginning in 1967, India reserved the production of certain products for small-scale industry (SSI) units. These reservations were gradually eliminated between 1997 and 2015 (Martin et al., 2017; Boehm et al., 2022). This "dereservation" was implemented in a staggered manner across product categories. If distortions lower the rate of adoption of technologies such as ERP systems or software, firms in dereserved categories should exhibit stronger scope economies. Table 3 presents regression results from the following specification:

$$\log p_{i,f,t} y_{i,f,t} = \beta \log n_{f,t} + \beta_{\text{der}} \log n_{f,t} \times \text{Dereserved}_{f,t} + \alpha_t + \alpha_i + \varepsilon_{i,f,t}.$$

This is estimated for products whose own category has not been dereserved in year t, so that the reform does not directly affect them. $\text{Dereserved}_{f,t}$ equals one if the firm has a product in a dereserved category in year t. Column (2) shows that the slope is negative (-0.196) for firms without a dereserved product but 0.568 higher for firms with one. Therefore, distortions also act on the scope margin, possibly through limited adoption of non-rival technologies.

Table 3: Revenue per Product and Scope: Dereservation in India

	(1)	(2)
Log(Number of Products)	-0.046	-0.196***
	(0.042)	(0.040)
Log(Number of Products) × Dereserved firm		0.568***
		(0.077)
Observations	82373	82373
R^2	0.449	0.453

Notes: Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

3. Model

I introduce a heterogeneous-firm model with two key ingredients. First, firms choose the number of products, or scope. Second, firms choose whether to adopt non-rival management. Non-rivalry here is *within* the firm, so non-rival management is shared across its products. A representative

[12] See Section 3 for how the model captures this pattern.

household supplies labor and owns all the firms. A competitive final-good firm combines intermediate goods produced by a unit continuum of monopolistically competitive intermediate-good firms. There is no aggregate uncertainty.

3.1. Environment

The representative household has utility $U(C) = C$, supplies $L = 1$ units of labor inelastically at wage W, and collects all profits. Intermediate-good firms are indexed by f. Each firm produces a continuum of differentiated products $i \in [0, n_f]$, where n_f is the scope, or number of products, of the firm. The final-good firm combines intermediate goods into the final good

$$Y = \left(\int \left\{ \int_0^{n_f} y_{i,f}^{\frac{\sigma-1}{\sigma}} \, di \right\} \, df \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$ is the elasticity of substitution between products. Cost minimization implies the demand curve

$$y_{i,f} = \left(\frac{p_{i,f}}{P} \right)^{-\sigma} Y \tag{2}$$

with an aggregate price index

$$P = \left(\int \left\{ \int_0^{n_f} p_{i,f}^{1-\sigma} \, di \right\} \, df \right)^{\frac{1}{1-\sigma}}, \tag{3}$$

which is the price of the final good. Products within and across firms are equally substitutable.

3.2. Technology

Firms produce output $y_{i,f}$ of product $i \in [0, n_f]$ using a constant-returns-to-scale technology:

$$y_{i,f} = z_{i,f} \ell_{i,f}^{1-\alpha-\gamma} x_{i,f}^{\gamma} m_{i,f}^{\alpha}, \qquad \alpha, \gamma \in (0,1), \ \alpha + \gamma < 1.$$

This technology combines three inputs: manual labor $\ell_{i,f}$, materials $x_{i,f}$, and the managerial input $m_{i,f}$. Each product has its own productivity $z_{i,f}$. The parameter α is the share of the managerial input and γ the materials share. Manual labor costs the wage W and materials cost P.

Production across products is linked through the potentially non-rival nature of management. I assume that manual labor and materials are rival. That is, a firm must replicate them for each product. The managerial input depends on the firm's organization, which I denote by $d_f \in \{0, 1\}$. I interpret the managerial input as the manager's effective time, and d_f selects whether this time is rival or non-rival across products. Under rival management ($d_f = 0$), the manager's time is composed of labor alone and requires replication across products. Under non-rival management ($d_f = 1$), the manager's time is augmented by organization capital, such as

software, and partly or fully shared across products. Adopting non-rival management requires a fixed cost of f_e units of the final good, so firms do not always adopt it. The paragraphs below formalize each input in turn.

Labor and Materials. Define ℓ_f and x_f as the total manual labor and materials used by the firm, with associated costs $W\ell_f$ and Px_f. Allocations across products are restricted:

$$\int_0^{n_f} \ell_{i,f}\, di \leq \ell_f, \qquad \int_0^{n_f} x_{i,f}\, di \leq x_f.$$

Rival Management ($d_f = 0$). The firm's total managerial input is m_f, and allocations across products are restricted:

$$\int_0^{n_f} m_{i,f}\, di \leq m_f.$$

Managerial time is labor, $m_f = \ell_f^m$, drawn from the same pool as manual labor, so its price is $P_m^0 = W$.

Non-Rival Management ($d_f = 1$). Here, the manager's effective time across products is subject to

$$\left(\int_0^{n_f} m_{i,f}^{\frac{1}{1-\rho}}\, di\right)^{1-\rho} \leq m_f, \qquad \rho \in [0,1].$$

The parameter ρ captures the strength of non-rivalry. It spans the cases from fully rival ($\rho = 0$, where the constraint collapses to the rival one) to fully non-rival ($\rho = 1$, where $\max_i m_{i,f} \leq m_f$), following Crouzet et al. (2026). Equivalently, it measures how complementary the manager's time is across products, and $\rho = 1$ means perfect complementarity. The firm-level managerial input is a Cobb–Douglas combination of managerial labor ℓ_f^m and organization capital s_f^m,

$$m_f = \left(\ell_f^m\right)^{1-\phi} \left(s_f^m\right)^{\phi}, \qquad \phi \in [0,1], \tag{4}$$

where organization capital fully depreciates and is produced one-for-one from the final good, so its rental rate is P. The price of the managerial input under adoption is therefore[13]

$$P_m^1 = \left(\frac{W}{1-\phi}\right)^{1-\phi} \left(\frac{P}{\phi}\right)^{\phi}. \tag{5}$$

With $\phi > 0$, the distortions introduced in Section 6 increase the relative cost of non-rival management, consistent with the pattern Hjort et al. (2026) document.

[13] P_m^1 is the minimum cost of one unit of m_f: $P_m^1 = \min_{\ell_f^m, s_f^m} W\ell_f^m + Ps_f^m$ subject to $(\ell_f^m)^{1-\phi}(s_f^m)^{\phi} = 1$. As $\phi \to 0$, $P_m^1 \to W$, the price under rival management.

Productivity. Let z_f be the firm's core productivity. Following Arkolakis et al. (2021), product-specific productivities decline with product rank,

$$\log z_{i,f} = \log z_f - \delta \log(i+1), \qquad \delta \geq 0, \qquad z_f \sim \text{Pareto}(\zeta). \tag{6}$$

Product $i = 0$ is the core product, and δ captures core competency: products further from the core are less productive. I introduce core competency to match the variation in sales within firms. It is not needed for the amplification results.[14]

Scope Costs. Firms pay a cost, in units of the final good, to maintain n_f products,

$$\mathcal{C}_f(n_f) = f_{c,f} \frac{n_f^{1+\omega}}{1+\omega}, \qquad \omega > 0, \qquad \log f_{c,f} \sim N\left(\mu_c, \sigma_c^2\right) \text{ i.i.d. across firms, independent of } z_f. \tag{7}$$

Heterogeneity in $f_{c,f}$ is necessary so that, for a given organization choice, the slope of log revenue per product on the log number of products is not driven by productivity alone. Like core competency, this heterogeneity serves to match the data.[15]

3.3. Firm's Problem

I characterize the firm's choices in two steps. First, conditional on the firm's organization, $d_f = d$, the firm's profits Π_f^d are the value of the following problem:

$$\Pi_f^d = \max_{\substack{n_f, m_f, \\ \{\ell_{i,f}, x_{i,f}, m_{i,f}\}_i \\ \{y_{i,f}, p_{i,f}\}_i}} \int_0^{n_f} p_{i,f} y_{i,f} \, di - W \int_0^{n_f} \ell_{i,f} \, di - P \int_0^{n_f} x_{i,f} \, di - P_m^d m_f - \mathcal{C}_f(n_f) \tag{8}$$

$$\text{subject to} \quad y_{i,f} = (p_{i,f}/P)^{-\sigma} Y, \tag{9}$$

$$y_{i,f} = z_{i,f} \ell_{i,f}^{1-\alpha-\gamma} x_{i,f}^{\gamma} m_{i,f}^{\alpha}, \tag{10}$$

$$\text{if } d=0: \quad \int_0^{n_f} m_{i,f} \, di = m_f, \qquad \text{if } d=1: \quad \left(\int_0^{n_f} m_{i,f}^{\frac{1}{1-\rho}} \, di \right)^{1-\rho} = m_f. \tag{11}$$

The firm chooses scope, prices, output, and input allocations across products. Its profits are total revenues across all its products net of the costs of labor, materials, the managerial input, and scope maintenance $\mathcal{C}_f(n_f)$. These choices are subject to demand (9) and the production technology (10) for each product. They are also subject to the constraint on allocating managerial time, which I term the *organizational constraint* (11).

Second, the firm adopts if its profits net of the adoption cost f_e are at least as large as its

[14] Amplification relies on adoption thickening the tail of the firm-size distribution; this occurs with $\delta > 0$ or $\delta = 0$, unless core competency is strong (Proposition 4).

[15] Theoretical results are derived with $\sigma_c = 0$ and confirmed by quantitative results with a calibrated value for σ_c (Table 5).

profits without adoption:

$$d_f = \begin{cases} 1 & \text{if } \Pi_f^1 - f_e \geq \Pi_f^0, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Adoption follows a threshold rule in z_f, with a threshold that varies with $f_{c,f}$.

3.4. Equilibrium

The final good is the numeraire, $P = 1$. An equilibrium consists of a wage W, aggregates (Y, C), and, for each firm f, policies d_f, n_f, m_f, ℓ_f^m, s_f^m, and $\{\ell_{i,f}, x_{i,f}, m_{i,f}, y_{i,f}, p_{i,f}\}_i$ such that:

(i) **Household:** the household supplies $L = 1$ inelastically and consumes $C = W + \Pi$, where $\Pi = \int \left(\Pi_f^{d_f} - f_e d_f\right) df$ denotes aggregate profits net of adoption costs.

(ii) **Final-good firm:** taking $\{p_{i,f}\}_{i,f}$ as given, the final-good firm minimizes the cost of producing Y in (1), which yields the demand (2).

(iii) **Intermediate-good firms:** taking W and Y as given, each firm's policies solve problem (8)–(11), with $m_f = (\ell_f^m)^{1-\phi d_f}(s_f^m)^{\phi d_f}$, $P_m^0 = W$, P_m^1 given by (5), and d_f given by (12).

(iv) **Labor market:** $\int \ell_f \, df + \int \ell_f^m \, df = 1$.

(v) **Final-good market:** $Y = C + \int x_f \, df + \int s_f^m \, df + \int \mathcal{C}_f(n_f) \, df + f_e \int d_f \, df$.

4. Model Characterization

In this section, I show that the multi-product firm's problem (equation (8)) reduces to a single-product representation in which scope economies appear in firm-level productivity. I use this representation to analyze the scope and adoption decisions and characterize the implications of scope economies for the firm-size and scope distributions.

4.1. Firm Decisions

I solve the firm's problem in two steps. Given d_f, n_f, and firm-level inputs (ℓ_f, x_f, m_f), the firm first allocates these inputs across its products. Substituting the optimal within-firm allocations leaves a choice of n_f and (ℓ_f, x_f, m_f) alone. This problem is that of a single-product firm whose productivity depends on its scope and on how it organizes production. The following proposition states this representation.

Proposition 1. *Let* $A \equiv PY^{\frac{1}{\sigma}}$ *and* $\eta \equiv \frac{\sigma-1}{\sigma}$. *Conditional on* $d_f = d$, *the firm's profits* Π_f^d *are the value of the following problem:*

$$\Pi_f^d = \max_{n_f, \ell_f, x_f, m_f} A\left(Z_f \ell_f^{1-\alpha-\gamma} x_f^{\gamma} m_f^{\alpha}\right)^{\eta} - W\ell_f - Px_f - P_m^d m_f - \mathcal{C}_f(n_f) \tag{13}$$

$$\text{where} \quad \text{if } d=0\text{:} \quad Z_f = \left(\int_0^{n_f} z_{i,f}^{\frac{\eta}{1-\eta}}\, di\right)^{\frac{1-\eta}{\eta}},$$

$$\text{if } d=1\text{:} \quad Z_f = \left(\int_0^{n_f} z_{i,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\, di\right)^{\frac{1-\eta(1-\rho\alpha)}{\eta}}.$$

The proof is in Appendix B.1. In this representation, revenues are Ay_f^{η}, with $y_f \equiv Z_f \ell_f^{1-\alpha-\gamma} x_f^{\gamma} m_f^{\alpha}$ the firm's output. This firm-level technology has the same form as the product-level technology. The representation maps to existing single-product models, such as Lucas (1978), where η is the span-of-control parameter, or Melitz (2003). What differentiates this model is the firm-level productivity, Z_f, which aggregates the product-level productivities $\{z_{i,f}\}_i$ and responds to the firm's choices of scope and organization.[16]

The adoption choice, $d_f \in \{0, 1\}$, remains as stated in equation (12). Choosing to use non-rival management changes how the firm aggregates $\{z_{i,f}\}_i$ into Z_f. Hence, the choice resembles technology adoption, except that the productivity jump comes from how the firm organizes production rather than from acquiring a frontier technology. Firm-level productivity Z_f depends on scope, n_f, which changes how many products the firm aggregates across, and on organization, d_f, which changes how it aggregates them. This matches the sources of scope economies discussed in Section 2.4.

Proposition 1 is in the spirit of Acemoglu and Restrepo (2022), who represent output in a task-based economy as a CES aggregate with endogenous task shares and trace the effects of automation through these shares. Engbom et al. (2025) obtain a similar reduction within the firm: an entrepreneur's assignment of administrative tasks reduces to a Cobb–Douglas problem in which the firm's organization determines its productivity, factor shares, and returns to scale. Here as well, the firm's organization determines its productivity. I trace the gains from better organization to the scope margin. I use this representation to study size-dependent distortions (Section 6).

[16] When $d_f = 0$, Z_f shares the same functional form as aggregate productivity across firms in environments with CES demand and single-product firms (Hopenhayn, 2014).

Firm-Level Factor Demand. The first-order conditions of problem (13) for ℓ_f, x_f, and m_f are

$$\begin{aligned}
\ell_f &= (A\eta/\Omega_d)^{\frac{1}{1-\eta}}\,\Omega_d \frac{(1-\alpha-\gamma)}{W} Z_f^{\frac{\eta}{1-\eta}} \\
x_f &= (A\eta/\Omega_d)^{\frac{1}{1-\eta}}\,\Omega_d \frac{\gamma}{P} Z_f^{\frac{\eta}{1-\eta}} \\
m_f &= (A\eta/\Omega_d)^{\frac{1}{1-\eta}}\,\Omega_d \frac{\alpha}{P_m^d} Z_f^{\frac{\eta}{1-\eta}},
\end{aligned}$$

where $\Omega_d = \left(\frac{W}{1-\alpha-\gamma}\right)^{(1-\alpha-\gamma)} \left(\frac{P}{\gamma}\right)^{\gamma} \left(\frac{P_m^d}{\alpha}\right)^{\alpha}$ is the unit cost function associated with the product-level technology. If $\phi > 0$, then $\Omega_1 \neq \Omega_0$ because of the organization capital used to produce the managerial input. These allocations are analogous to those in a single-product environment with firm-level productivity Z_f.

Within-Firm Allocations. With the introduction of firm-level productivity Z_f, within-firm allocations are conveniently functions of $z_{i,f}$ and Z_f. In particular, if $d_f = 0$, within-firm allocations relative to firm totals are

$$\frac{\ell_{i,f}}{\ell_f} = \frac{x_{i,f}}{x_f} = \frac{m_{i,f}}{m_f} = \frac{p_{i,f}y_{i,f}}{\int_0^{n_f} p_{i,f}y_{i,f}\,di} = \left(\frac{z_{i,f}}{Z_f}\right)^{\frac{\eta}{1-\eta}}. \tag{14}$$

If $d_f = 1$, they are

$$\frac{\ell_{i,f}}{\ell_f} = \frac{x_{i,f}}{x_f} = \frac{p_{i,f}y_{i,f}}{\int_0^{n_f} p_{i,f}y_{i,f}\,di} = \left(\frac{z_{i,f}}{Z_f}\right)^{\frac{\eta}{1-\eta(1-\rho\alpha)}}, \qquad \frac{m_{i,f}}{m_f} = \left(\frac{z_{i,f}}{Z_f}\right)^{\frac{\eta(1-\rho)}{1-\eta(1-\rho\alpha)}}. \tag{15}$$

When firms adopt non-rival management, both exponents fall relative to $d_f = 0$. The effective time of the manager is more evenly shared across products, and sales are less concentrated within the firm.

4.2. Scope Economies

In this subsection, I illustrate how scope, n_f, and organization, d_f, affect firm-level productivity, Z_f. To understand the role of scope economies, consider the elasticity of firm-level productivity with respect to scope, fixing $\delta = 0$ for exposition:

$$\frac{\partial \log Z_f}{\partial \log n_f} = \underbrace{\frac{1-\eta}{\eta}}_{\text{love of variety}} + \underbrace{d_f \cdot \rho\alpha}_{\text{scope economies}}. \tag{16}$$

The elasticity shows that as firms increase the number of products they offer, their productivity increases at the love-of-variety rate, $\frac{1-\eta}{\eta} = \frac{1}{\sigma-1}$ (Matsuyama and Ushchev, 2025).[17] This is

[17] Appendix B.1.2 derives this expression for arbitrary $\{z_{i,f}\}_i$.

a useful benchmark because it is also the marginal utility of new firms in a single-product CES economy (Hopenhayn, 2014). With scope economies, the marginal benefit of new varieties increases beyond love of variety: if the firm adopts non-rival management ($d_f = 1$), the elasticity increases by $\rho\alpha$. The stronger the non-rivalry (the higher ρ), the greater the effect of n_f on Z_f. This creates a supply-side motive to introduce new varieties into the economy.

Holding n_f fixed, let Z_f^d denote firm-level productivity under $d_f = d$, and let $\Delta \log Z_f \equiv \log Z_f^1 - \log Z_f^0$. Under symmetry ($\delta = 0$), $\Delta \log Z_f = \rho\alpha \log n_f$. Adoption of non-rival management therefore appears as a jump in firm-level productivity, $Z_f^1/Z_f^0 = n_f^{\rho\alpha}$, which is larger for firms with broader scope and increases with ρ.[18]

Linking Productivity to Cost Sharing. I now link productivity explicitly to the cost savings generated by non-rival management, by characterizing the firm's cost minimization conditional on n_f and d_f.

Proposition 2. *Given n_f, d_f, and output levels $\{y_{j,f}\}_j$, the marginal cost of producing $y_{i,f}$ is*

$$MC_{i,f} = \begin{cases} \dfrac{\Omega_0}{z_{i,f}} & \text{if } d_f = 0, \\ \dfrac{\Omega_1}{z_{i,f}} \left(\dfrac{\left(\frac{y_{i,f}}{z_{i,f}}\right)^{\frac{1}{1-\rho\alpha}}}{\int_0^{n_f} \left(\frac{y_{j,f}}{z_{j,f}}\right)^{\frac{1}{1-\rho\alpha}} dj} \right)^{\rho\alpha} & \text{if } d_f = 1, \end{cases} \tag{17}$$

where Ω_d is the unit cost under organization choice d (defined in Section 4.1).

Proposition 2 characterizes the marginal cost of producing $y_{i,f}$. If $d_f = 0$, production is independent across products, so marginal cost is the familiar constant-returns-to-scale marginal cost, equal to the unit cost divided by productivity. If $d_f = 1$, marginal cost also depends on the output of the other products, because products share the managerial input. In this case, the unit cost is scaled by an additional term that depends on the sales share of product i (equation (15)). Substituting the optimal output $y_{i,f}$, marginal cost when $d_f = 1$ is

$$MC_{i,f} = \frac{\Omega_1}{z_{i,f}} \left(\frac{z_{i,f}}{Z_f} \right)^{\frac{\rho\alpha\eta}{1-\eta(1-\rho\alpha)}}.$$

Marginal cost is lower than in the single-product counterfactual, in which this term equals one. Its elasticity with respect to the product's sales share is $\rho\alpha$. Holding $z_{i,f}$ fixed, a higher Z_f lowers marginal cost. Under symmetry ($\delta = 0$),

$$MC_{i,f} = \frac{\Omega_1}{z_f} n_f^{-\rho\alpha}.$$

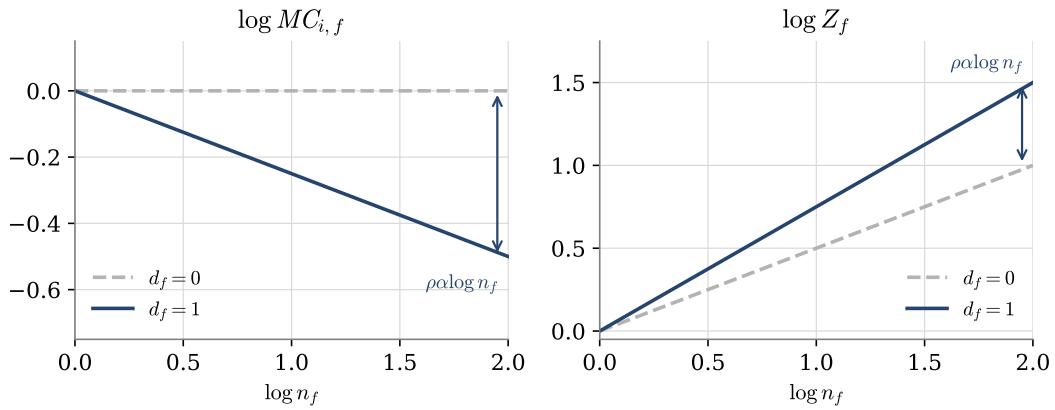
[18] Appendix B.1.3 derives an analogous expression for arbitrary $\{z_{i,f}\}_i$, of which this term is the first-order component.

When a firm adopts non-rival management, scope economies arise as cost savings for existing products. The parameter ρ, the strength of non-rivalry, governs the speed at which adding products lowers marginal cost. Define $\Delta \log MC_{i,f}$ analogously to $\Delta \log Z_f$, as the difference in log marginal cost between $d_f = 1$ and $d_f = 0$, holding n_f fixed:

$$\Delta \log MC_{i,f} = \underbrace{-\Delta \log Z_f}_{\text{cost savings}} - \underbrace{(\log \Omega_0 - \log \Omega_1)}_{\text{cost advantage}}.$$

The difference in marginal cost between the two organization choices can be decomposed into two terms. The first term, cost savings, is the productivity-enhancing role of non-rival management; the second, the cost advantage, reflects differences in input prices across the two technologies. The cost-advantage term is taken as given by the firm, but the cost-savings term is endogenous to the scope of the firm. This decomposition is generalized to arbitrary $\{z_{i,f}\}_i$ in Appendix B.1.5. Figure 4 plots $MC_{i,f}$ and Z_f against n_f for each d_f. The productivity jump stems from cost savings due to non-rivalry.

Figure 4: Marginal Cost and Productivity in Scope



Notes: Log marginal cost of product i and log firm-level productivity against log scope, holding core productivity fixed, under symmetric products ($\delta = 0$) and a common unit cost ($\Omega_1 = \Omega_0$). The dashed line is the policy under rival management ($d_f = 0$) and the solid line that under non-rival management ($d_f = 1$). In both panels the gap between the two is $\rho\alpha \log n_f$. The figure is illustrative, with $\frac{1-\eta}{\eta} = 0.5$ and $\rho\alpha = 0.25$.

4.3. Scope and Organization

Thus far, I have taken scope and organization as given. In this subsection, I characterize how the firm chooses them and what these choices imply for returns to scale and firm-level productivity.

Scope. The first-order condition of problem (13) with respect to n_f equates the marginal cost of scope to the marginal benefit,

$$\underbrace{f_{c,f} n_f^{\omega}}_{\text{marginal cost}} = \underbrace{(1-\eta) p_{n,f} y_{n,f}}_{\text{profit on product } n} + \underbrace{d_f \cdot \rho\alpha\eta \, p_{n,f} y_{n,f}}_{\text{managerial cost saved by sharing}} . \tag{18}$$

The first term is the profit on the marginal product. This is because $(1-\eta)$ is the profit rate and $p_{n,f} y_{n,f}$ is the revenue from this marginal product. The second term is active for firms that adopt non-rival management ($d_f = 1$). This term scales the revenue of the marginal product by $\rho\alpha\eta$, reflecting the cost savings from sharing the non-rival managerial input with a new product. Recall that marginal cost falls at a rate of $\rho\alpha$ with respect to scope. This cost-saving motive therefore means that adopters choose greater scope than their non-adopting counterparts.[19]

To obtain closed-form expressions, I assume that productivities are symmetric within the firm ($\delta = 0$), that firms share a common scope-maintenance cost ($\sigma_c = 0$, so $f_{c,f} = f_c$), and that adoption is costless ($f_e = 0$). Appendix B.1.6 derives all expressions below. In this simplified model, equation (18) gives

$$n_f = \left(\frac{(1 - \eta(1 - d_f \cdot \rho\alpha)) \, A^{\frac{1}{1-\eta}}}{f_c} \left(\frac{\eta z_f}{\Omega_{d_f}} \right)^{\frac{\eta}{1-\eta}} \right)^{\frac{1-\eta}{(1-\eta)\omega - d_f \cdot \rho\alpha\eta}} . \tag{19}$$

Observe that non-rival management makes scope more responsive to core productivity:

$$\frac{\partial \log n_f}{\partial \log z_f} = \frac{\eta}{(1-\eta)\omega - d_f \cdot \rho\alpha\eta}. \tag{20}$$

Both expressions require the parameter restriction $\omega > \frac{\rho\alpha\eta}{1-\eta}$, which I maintain throughout.[20] By equation (20), non-rival management widens scope differences across firms, which shapes the firm-size distribution (Section 4.4).

Returns to Scale. Non-rival management acts primarily on scope economies, as the first-order condition for n_f makes clear (equation (18)), but it also has implications for returns to scale. Recall that revenues are $A y_f^{\eta}$ (Proposition 1). In this equivalent setup, I ask what happens when the firm scales up its composite input, $F_f \equiv \ell_f^{1-\alpha-\gamma} x_f^{\gamma} m_f^{\alpha}$. In the simplified model,

$$\text{RTS}_f \equiv \frac{d \log y_f}{d \log F_f} = \frac{1}{1 - \frac{\eta}{1+\omega} \left[\frac{1-\eta}{\eta} + d_f \cdot \rho\alpha \right]}.$$

The term in brackets is the elasticity of Z_f with respect to n_f (equation (16)). Even when $d_f = 0$, returns to scale exceed one. If F_f rises, so does the revenue of the marginal product, so

[19] This observation is consistent with the data on organization capital in Section 2.4.

[20] The marginal cost of scope must rise faster than the marginal benefit; otherwise, scope is unbounded for adopting firms. The same restriction makes profits under adoption positive.

the firm adds products (equation (18)), which raises Z_f and hence y_f. Because this elasticity is greater when $d_f = 1$, returns to scale are also higher, so firm organization affects them, as in Engbom et al. (2025).

Adoption. The analysis of scope above takes d_f as given. I now turn to the adoption decision, which follows a threshold rule. Let $n_f^d(z)$ and $Z_f^d(z)$ denote scope and firm-level productivity under $d_f = d$ for a firm with core productivity z. I suppress the argument when it equals the firm's own z_f.

Proposition 3. *In the simplified model, $d_f = 1$ if and only if $z_f \geq \bar{z}$, or, equivalently, if and only if $n_f^0 \geq \bar{n}$, where n_f^0 is the scope the firm would choose without adopting and $\bar{n}$ is the value n_f^0 takes at $z_f = \bar{z}$. The threshold $\bar{z}$ decreases with the cost advantage of non-rival management,* $\log(\Omega_0/\Omega_1)$.

The cost advantage is $\Omega_0/\Omega_1 = \left(P_m^1/W\right)^{-\alpha}$, and since $P_m^1 \propto W^{1-\phi}$, $\Omega_0/\Omega_1 \propto W^{\alpha\phi}$. Wages therefore move adoption only through the share of organization capital in the managerial input, ϕ, and not at all when $\phi = 0$. Proposition 3 states that adopting non-rival management is optimal only for firms with enough products to share it across (Section 4.2).[21] I make two observations about firms around the adoption threshold. At the threshold, scope and firm-level productivity jump discretely, and away from it, the gain from adoption rises with core productivity and hence with firm size.

At the Threshold. Scope and firm-level productivity jump by

$$\log \frac{n_f^1(\bar{z})}{\bar{n}} = \frac{1}{1+\omega} \log \left(\frac{\omega\,(1+g)}{\omega - g} \right),$$
$$\log \frac{Z_f^1(\bar{z})}{\bar{Z}} = \frac{1-\eta}{\eta} \log \frac{n_f^1(\bar{z})}{\bar{n}} + \rho\alpha \log n_f^1(\bar{z}),$$

where $\bar{Z} = Z_f^0(\bar{z})$ is firm-level productivity at the threshold without adoption and $g = \frac{\rho\alpha\eta}{1-\eta}$, which increases with the strength of non-rivalry, ρ. The jump in scope depends only on ω and g, so stronger scope economies widen the gap in the scope distribution. The productivity jump has two terms. The first is a love-of-variety term, reflecting the value of the products the firm adds by adopting at the threshold. The second is a scope-economies term, which scales the adopter's log scope by $\rho\alpha$, since an adopting firm's productivity rises with scope at the additional rate $\rho\alpha$.

[21] The threshold is stated in units of n_f, so the normalization of a product sets the level of $\bar{n}$. If adoption carries no price premium ($\Omega_1 = \Omega_0$), then $\bar{n} < 1$, and with $f_e = 0$ every firm with counterfactual scope above one adopts; non-adopters therefore require $\Omega_1 > \Omega_0$ or $f_e > 0$. In that case $\log n_f^1(\bar{z})$ can be negative, so the second term of the productivity jump below need not be positive. Neither case arises in the calibration, where $\bar{n} > 1$.

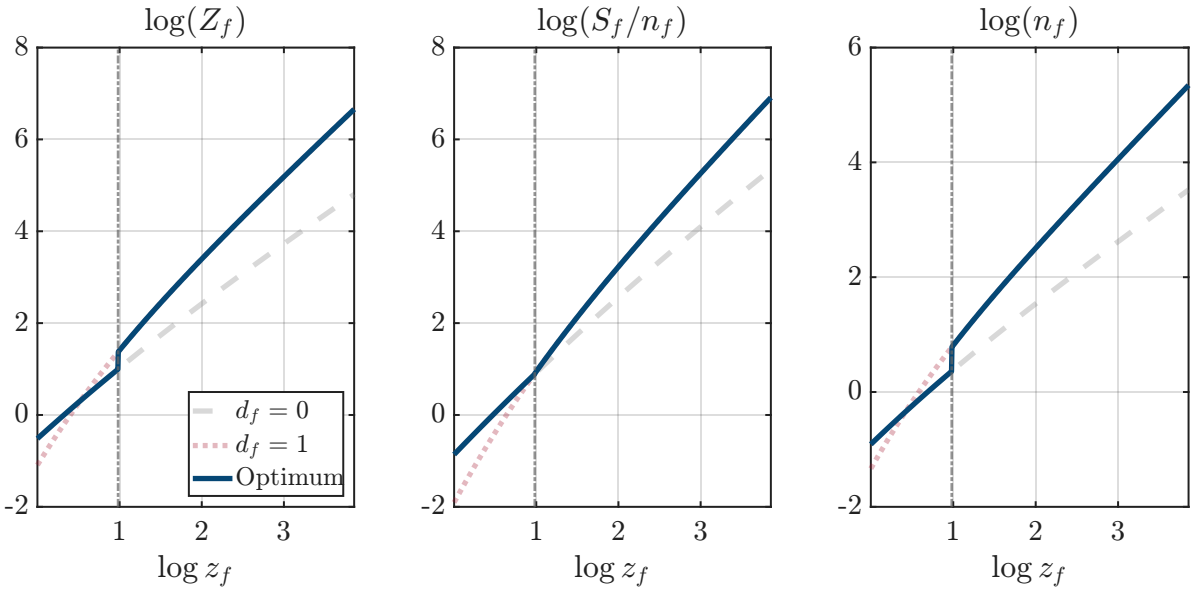
Away from the Threshold. The gain from adoption rises with core productivity at the rate

$$\frac{\partial}{\partial \log z_f} \log \frac{Z_f^1}{Z_f^0} = \frac{1+\omega}{\omega} \frac{g}{\omega - g} > 0. \tag{21}$$

This rate increases with ρ, the strength of non-rivalry.

Figure 5 brings together the firm's choices in the calibrated model. It plots firm-level productivity Z_f, revenue per product S_f/n_f, and scope n_f against z_f, for both $d_f = 0$ and $d_f = 1$, where $S_f = \int_0^{n_f} p_{i,f} y_{i,f}\, di$ denotes the sales of firm f. The vertical line marks the adoption threshold. Firm-level productivity and scope both jump at the threshold, and their gaps between the two organization choices widen with z_f.

Figure 5: Firm Policies



Notes: All variables are in logs. The scope-maintenance cost $f_{c,f}$ is fixed at its median value.

4.4. Firm-Size Distribution

Non-rival management raises firm-level productivity, with gains that grow with core productivity (equation (21)). Firm organization therefore affects how firm size scales with core productivity, which shapes the right tail of the firm-size distribution.

Recall that core productivity is Pareto, $\mathbb{P}(z_f > z) = (z_{\min}/z)^{\zeta}$. Under the simplifying assumptions of Section 4.3, scope is a power of z_f for each d_f (equation (19)). Since a power of a Pareto variable is also Pareto, the tail index of scope is ζ divided by the elasticity in equation (20).[22] Let $\zeta_n^{(d)}$ be the tail index of the distribution of n_f if all firms choose $d_f = d$.

[22] I define the tail index as $-\lim_{x\to\infty} \frac{d \log \mathbb{P}(n_f > x)}{d \log x}$.

Then

$$\zeta_n^{(0)} = \zeta \frac{1-\eta}{\eta} \omega, \qquad \zeta_n^{(1)} = \zeta \left(\frac{1-\eta}{\eta} \omega - \rho\alpha \right), \qquad \zeta_n^{(0)} - \zeta_n^{(1)} = \zeta\rho\alpha. \tag{22}$$

Because adoption raises the elasticity of scope to core productivity (equation (20)), more productive firms expand scope by more, so the tail of the scope distribution is thicker under non-rival management. For each d_f, sales and employment are proportional to $n_f^{1+\omega}$. Their tail indices are therefore $\zeta_n^{(d)}/(1+\omega)$, and non-rival management lowers them by $\zeta\rho\alpha/(1+\omega)$.

Equation (22) assumes that all firms make the same choice. Adoption, however, follows a threshold rule (Proposition 3). The firms that adopt are those with the highest core productivity, and hence the broadest scope. If a positive measure of firms adopts, adopters therefore occupy the tail, and the tail index of the economy is $\zeta_n^{(1)}$. Non-rival management thus thickens the tails of the scope and firm-size distributions, all else equal. This is consistent with the data. The US, where scope economies are present, has thicker tails in both distributions than India (Figures 1a and 1b). The following proposition shows that these results hold in the full model.

Proposition 4. *Suppose $\rho\alpha > 0$, $\omega > \frac{\rho\alpha\eta}{1-\eta}$, and $\frac{\delta\eta}{1-\eta} < 1$. Then, if a positive measure of firms adopts, adopters occupy the tail, $\mathbb{P}(d_f = 1 \mid n_f > x) \to 1$ as $x \to \infty$, and scope has tail index*

$$\zeta_n^{(1)} = \zeta \left(\frac{1-\eta}{\eta} \omega - \rho\alpha + \delta \right). \tag{23}$$

Sales and employment have tail index $\zeta_n^{(1)}/(1+\omega)$. In an economy in which no firm adopts, the tail index of scope is $\zeta_n^{(0)} = \zeta \left(\frac{1-\eta}{\eta} \omega + \delta \right)$. Both expressions reduce to equation (22) when $\delta = 0$. Hence $\zeta_n^{(0)} - \zeta_n^{(1)} = \zeta\rho\alpha$, independent of δ, the distribution of $f_{c,f}$, f_e, and Ω_0/Ω_1.

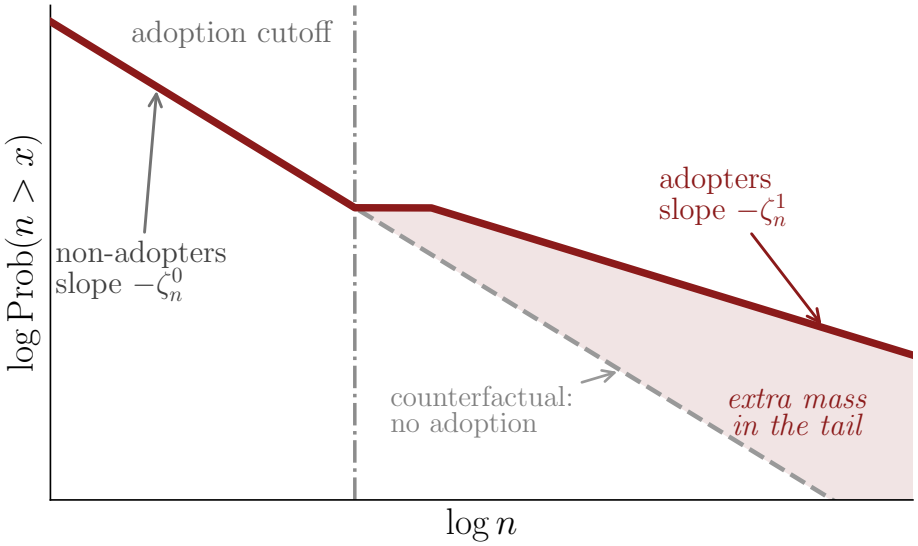
The proof is in Appendix B.2. Core competency (δ) thins both tails by the same amount, $\zeta\delta$. Heterogeneity in scope costs does not affect either tail, because the log-normal $f_{c,f}$ has thinner tails than any Pareto distribution, so the tail is determined by z_f. The thickening of the tail is governed by $\rho\alpha$ alone. The costs of adoption, f_e and Ω_0/Ω_1, determine which firms adopt but not the slope of the tail.

This change in the tail index distinguishes the model from standard models of technology adoption, in which adopters pay a fixed cost to raise productivity by a common proportion (e.g., Buera et al., 2026). A proportional gain increases the size of every adopter by the same factor, so it shifts the tail of the firm-size distribution but leaves its tail index unchanged. Non-rival management changes only how the firm is organized, yet it tilts the tail of the firm-size distribution rather than shifting it.

Figure 6 shows the entire scope distribution, not only its tail, in the simplified model on a log-log scale (Appendix B.2 derives it). The complementary CDF has three segments. Among non-adopters, it has slope $-\zeta_n^{(0)}$. Since scope jumps at the adoption threshold, no firm has scope between $\bar{n}$ and $n_f^1(\bar{z})$, and the complementary CDF is flat over this range, at a height equal to the share of firms that adopt. Among adopters, it has the flatter slope $-\zeta_n^{(1)}$. Non-rivalry sets

the slope of the adopter segment, while the adoption threshold sets where that segment begins, and hence the share of firms on it. The shaded area is the additional mass of high-scope firms relative to an economy in which no firm adopts.

Figure 6: Scope Distribution in the Simplified Model



Notes: Complementary CDF of scope, $\mathbb{P}(n_f > x)$, on a log-log scale, from equation (B.2). The dash-dotted line marks the adoption threshold $\bar{n}$. The dashed line is the distribution if no firm adopts non-rival management.

Implications for Calibration. Appendix B.2 derives two further properties. First, the length of the flat segment, $\log\left(n_f^1(\bar{z})/\bar{n}\right)$, increases with ρ. The model therefore links the size of a "missing middle" in the firm-size distribution to the strength of non-rivalry; its presence in developing economies is debated (Hsieh and Olken, 2014). Second, the ratio of the tail indices of scope and sales, $1+\omega$, does not depend on ρ, so it is informative about the convexity of scope costs. Section 5.3 uses these properties to discipline the model.

5. Identification and Calibration

In this section, I discuss how to discipline the model. I first show the shortcomings of using the elasticity of revenue per product with respect to the number of products in the cross section. Then I propose an instrumental-variable (IV) strategy to identify this elasticity. I map the estimated elasticity to model parameters and use it, together with the firm-size distribution, to discipline the strength of non-rivalry, ρ, and the cost of adoption, f_e. I also describe how to discipline the remaining parameters. I find that 35% of firms adopt non-rival management and that $\rho = 0.818$, close to full non-rivalry.

5.1. Bias in the Cross Section

I begin by showing that the cross-sectional elasticity in Section 2.3 overstates scope economies. When δ is zero or very small, revenue per product can be expressed as a function of the number of products:

$$\log \frac{S_f}{n_f} = \log \left(A \left(A\eta / \Omega_{d_f} \right)^{\frac{\eta}{1-\eta}} \right) + d_f \cdot \frac{\rho\alpha\eta}{1-\eta} \log n_f + \frac{\eta}{1-\eta} \log z_f. \tag{24}$$

Holding z_f constant, $\frac{\partial \log \frac{S_f}{n_f}}{\partial \log n_f} = d_f \cdot \frac{\rho\alpha\eta}{1-\eta}$. Let $\bar{d}$ be the fraction of firms with $d_f = 1$. Then $\mathbb{E}\left(\frac{\partial \log \frac{S_f}{n_f}}{\partial \log n_f} \right) = \bar{d} \frac{\rho\alpha\eta}{1-\eta}$. The average elasticity of revenue per product with respect to the number of products therefore identifies the product of the strength of non-rivalry and the fraction of adopters, not each of them separately. Compare this with the regression analogue in the data:

$$\log \frac{S_{f,t}}{n_{f,t}} = \beta \log n_{f,t} + \psi_t + \varepsilon_{f,t}. \tag{25}$$

This regression is biased upward relative to the true average elasticity. Unobserved core productivity z_f generates two sources of upward bias: (1) along the intensive margin, conditional on d_f; and (2) along the extensive margin, by selecting for d_f. Isolating the intensive margin, suppose either all firms adopt or none do. Then

$$\beta_0 = \omega R^2_{z,0} \tag{26}$$

$$\beta_1 = \left(\omega - \frac{\rho\alpha\eta}{1-\eta} \right) R^2_{z,1} + \frac{\rho\alpha\eta}{1-\eta}, \tag{27}$$

where β_d is the coefficient from the regression when all firms choose $d_f = d$ and $R^2_{z,d}$ is the R^2 of a regression of $\log n_f$ on $\log z_f$ among those firms. Even if no firm adopts, β is positive, and the bias depends on how much of the variation in scope is driven by core productivity. If core productivity were the only source of that variation, then $R^2_{z,d} = 1$, the slope would equal ω under either organization choice, and $\frac{\rho\alpha\eta}{1-\eta}$ would not be identified. Heterogeneity in $f_{c,f}$ is therefore what makes the slope informative about scope economies.

When $\bar{d} \in (0, 1)$, the coefficient in specification (25) is approximately a weighted average of β_1 and β_0,

$$\beta \approx \bar{d}\, \beta_1 + \left(1 - \bar{d}\right) \beta_0. \tag{28}$$

This uses $\bar{d}(1 - \bar{d}) \approx 0$, which drops the terms carrying the extensive margin. Appendix B.3.1

derives the exact expression. The bias relative to the average elasticity is then

$$\text{Bias}(\beta) \equiv \beta - \mathbb{E}\left(\frac{\partial \log \frac{S_f}{n_f}}{\partial \log n_f}\right) \approx \bar{d}\left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right) R^2_{z,1} + \left(1-\bar{d}\right)\omega R^2_{z,0}. \tag{29}$$

The bias is positive since, under the maintained restriction, $\omega > \frac{\rho\alpha\eta}{1-\eta}$. The slope alone therefore cannot separate $\frac{\rho\alpha\eta}{1-\eta}$, ω, and $\bar{d}$, and is not a good candidate to identify the average elasticity of revenue per product with respect to the number of products. In Section 5.2, I instead instrument scope with input-cost shocks.

5.2. Identification Strategy

Identifying the elasticity of revenue per product with respect to the number of products requires variation in scope that is independent of core productivity z_f. In the data, firms' product portfolios span multiple sectors, so their input mix is heterogeneous. For example, a firm producing packaged food and paper products uses wheat for the former and wood pulp for the latter. A rise in the price of wheat reduces output in packaged food, and, if the rise is large enough, the profit on the marginal food line falls enough that the firm drops it. Conversely, a large fall in the price of wheat leads the firm to add food lines.

If the firm does not employ non-rival management, then dropping the food line has no effect on its paper products. Under non-rival management, the managerial input is shared across fewer products, which raises the marginal cost of each remaining product and reduces the revenue of the firm's paper products. Averaged across firms and scaled by the change in scope, this fall approximates the average elasticity of revenue per product with respect to the number of products, $\bar{d}\frac{\rho\alpha\eta}{1-\eta}$. I therefore use such cost shocks as an instrument for scope and ask how average revenues respond in sectors that are not exposed to them.

Construction. To implement this strategy, I construct a cost shock for each sector from the 2007 BEA detailed Use table for the US, which gives the cost share $\theta_{k,c}$ of each commodity c in each manufacturing sector k, at approximately the 3-digit NAICS level, and from BLS producer price indices $p_{c,t}$ for commodity c in year t:[23]

$$\text{costshock}_{k,t} = \sum_c \theta_{k,c} d\log p_{c,t}. \tag{30}$$

Next, I define the following set:

$$G_t = \{k : \text{costshock}_{k,t} > \bar{c} \quad \text{or} \quad \text{costshock}_{k,t} < \underline{c}\}. \tag{31}$$

[23] I use the BLS commodity (WP) and industry (PC) producer price index series, which I map by hand to the commodities of the 2007 BEA detailed Use table. I residualize each commodity's price-change series on quarter-of-year fixed effects and compute the annual change as the average of the residuals.

Given two thresholds, $\underline{c} < \bar{c}$, set at the 20th and 80th percentiles of the sector cost shocks, G_t is the set of exposed sectors in year t, that is, sectors whose input costs move sharply in either direction. The thresholds select shocks large enough that firms drop or add products in these sectors, as in the example above. Unexposed sectors, those outside G_t, have small but not necessarily zero shocks. I then map each product i to a sector $k(i)$ through its product group in NielsenIQ. Here, products are defined as UPCs rather than CMIE categories.[24] Let $s_{f,k}$ be the share of firm f's revenue in sector k, measured in one pre-period year and held fixed, with a two-year buffer before the estimation sample, so that it does not respond to the shocks themselves. I define the following firm-level cost-shock exposure measure:

$$B_{f,t} = \sum_{k \in G_t} s_{f,k} \text{costshock}_{k,t}. \tag{32}$$

This captures a firm's revenue-weighted exposure to large input-price shocks. A higher $B_{f,t}$ means higher input costs in the exposed sectors in which the firm operates, and hence fewer products. The estimation sample is the NielsenIQ sample described in Section 2.1, less the firms whose portfolios are too narrow to include both exposed and unexposed products.

Specification. To build the estimating equation, I average log product revenue over the firm's unexposed products. I then estimate the following regression:

$$\mathbb{E}\left(\log p_{i,f,t} y_{i,f,t} \mid k(i) \notin G_t\right) = \beta \log n_{f,t} + \psi_t + \varepsilon_{f,t}, \tag{33}$$

where I use $B_{f,t}$ as an instrument for $\log n_{f,t}$ and denote the resulting coefficient β_{IV}.

Mapping to the Model. Appendix B.3.2 extends the model to firms operating in several sectors, each with its own scope cost, where the firm pays P/a_k per unit of materials in sector k. A rise in a sector's materials cost is equivalent to a negative, sector-specific productivity shock. If a share s_f of the firm's sectors receives the shock $a_E < 1$, the exposure measure is $B_f = -s_f \gamma \eta \log a_E$. I characterize β_{IV} in the multi-sector extension (Proposition 8) and show how to equivalently recover the coefficient in a single-sector environment (Proposition 9). I summarize these results here:

Proposition 5. *Suppose $\delta = 0$, the exposed share s_f is independent of $(z_f, f_{c,f})$, the shock does not change d_f, and it is large enough that the firm's products in exposed sectors exit. Then:*

[24] CMIE categories serve to compare the US with India (Section 2.1), which a US-only design does not require, and they are too coarse for the instrument. The design needs firms that sell products in both exposed and unexposed sectors and whose scope can respond to the shocks. With CMIE categories, 62% of US firms sell a single product (Table 1) and so cannot have both exposed and unexposed products. With UPCs, 75% of firms sell more than one product (Appendix Table A2).

(i) The IV coefficient is

$$\beta_{IV} = \frac{\rho\alpha\eta}{1-\eta}\bar{d} \cdot \frac{\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}}{\bar{d}\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}} + 1 - \bar{d}}, \qquad \bar{d} = \mathbb{E}[d_f]. \tag{34}$$

(ii) The firm chooses the same scope as it would in the single-sector model with scope cost $(1 - s_f)^{-\omega} f_{c,f}$.

The assumptions are the exclusion and relevance conditions in terms of the model. Exclusion comes from the independence of s_f from $(z_f, f_{c,f})$. Relevance follows from the first stage, the response of $\log n_f$ to B_f. It is negative for non-adopters ($d_f = 0$) and larger in absolute value by the factor $\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}} > 1$ for adopters ($d_f = 1$).

Proposition 5 part (i) is the IV counterpart of equation (28). The IV estimator uses only variation in scope that comes from scope costs, so the term in ω that biases OLS upward drops out. What remains is $\frac{\rho\alpha\eta}{1-\eta}\bar{d}$ scaled by the ratio of adopters' first stage to the average first stage. This ratio exceeds one, so $\frac{\rho\alpha\eta}{1-\eta}\bar{d} < \beta_{IV} < \frac{\rho\alpha\eta}{1-\eta}$. Unlike the bias in OLS, this gap shrinks as ω rises. The IV estimator therefore puts the estimate closer to the average elasticity precisely where OLS is most biased.

As a consequence of part (ii), the first stage and the reduced form map into the single-sector model's elasticities of scope and revenue with respect to $f_{c,f}$. Therefore, β_{IV} can be computed in the calibrated model without solving the multi-sector model (Proposition 9). The same holds when $\delta > 0$ (Propositions 10 and 11). One can recover β_{IV} as the ratio between the average elasticity of revenue per product to $f_{c,f}$ and the average elasticity of n_f to $f_{c,f}$. In the calibrated model, β_{IV} is approximated well by $\frac{\rho\alpha\eta}{1-\eta}\bar{d}$.[25]

Results. Table 4 reports the estimates.[26] Firms with higher $B_{f,t}$ produce fewer products, and the first stage is strong (Appendix Tables A9 and A10). In the model, the reduced form is zero unless firms share a non-rival input, in which case it is negative. The estimated reduced-form coefficient is negative and significant at the 1% level, which is evidence of scope economies. The IV estimate is 0.476, against an OLS slope of 0.802. Since the IV estimate approximates the average elasticity, the gap between them is approximately the bias derived in Section 5.1. Instrumenting with exposure lagged one year, $B_{f,t-1}$, to address concerns about within-year financial frictions or inflexibility in adjusting scope, gives a smaller IV estimate, 0.200, that remains significant at the 1% level (column (3) of Table 4). I use this estimate, the more conservative of the two, in the calibration (Section 5.3).

[25] In the calibrated model, the exact β_{IV} and $\frac{\rho\alpha\eta}{1-\eta}\bar{d}$ differ by less than 0.01 in absolute value.

[26] The estimates are similar with symmetric thresholds at the 15th and 85th, or the 25th and 75th, percentiles of the sector cost shocks. These results are available on request.

Table 4: Revenue per Product and Scope: Cost-Shock Instrument

	Avg. log revenue		
	(1) OLS	(2) IV	(3) IV
Log # products	0.802***	0.476***	0.200***
	(0.010)	(0.100)	(0.069)
Instrument	—	$B_{f,t}$	$B_{f,t-1}$
Observations	139524	139524	132074
K-P F		354.09	562.63

Notes: K-P F is the Kleibergen–Paap first-stage F statistic. Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

5.3. Parameters and Targeted Moments

Assigned Parameters. I fix the materials share $\gamma = 0.6$ based on 2007 US data (BEA) and $\sigma = 3$ following Hsieh and Klenow (2009); hence, $\eta = 2/3$. I set $\alpha = 0.35$, the average share of SG&A in total costs in the Compustat annual sample of US firms. Lastly, I set $\phi = 0.3$ following Peters and Taylor (2017), who assume that 30% of SG&A is organization capital.

Calibrated Parameters. I calibrate the remaining parameters, reported in Table 5, to moments of the 2007 firm-size and scope distributions and the IV estimate (Table 6).[27] These are the parameters governing scope economies (non-rivalry ρ and the adoption cost f_e, which sets the adoption rate $\bar{d}$), the costs of scope (the curvature ω of the scope-maintenance cost and the mean μ_c and standard deviation σ_c of $\log f_{c,f}$), and technology (the Pareto shape ζ and core competency δ). All of them are calibrated jointly to match the targeted moments. Next, I explain which moments are most informative about which parameters.

Scope Economies. Recall from Section 5.2 that

$$\beta_{IV} \approx \mathbb{E}\left(\frac{\partial \log\left(S_f/n_f\right)}{\partial \log n_f}\right) = \bar{d}\frac{\rho\alpha\eta}{1-\eta}.$$

Since α and η are assigned, for each ρ I can back out the f_e that delivers the adoption rate $\bar{d}$ consistent with the IV estimate. In this approximate form, the estimate bounds both the adoption rate and non-rivalry. Since $\rho \leq 1$, it implies $\bar{d} \geq 0.29$, and, since $\bar{d} \leq 1$, $\rho \geq 0.29$.[28]

[27] Unlike the IV estimate, the scope moments define products as CMIE categories, the definition that the counterfactuals for India in Section 7 require. In the cross section, the elasticity of revenue per product with respect to the number of products is higher with CMIE categories than with UPCs. This suggests that the UPC-based IV estimate understates, rather than overstates, scope economies under the CMIE definition.

[28] By Proposition 5, β_{IV} exceeds the average elasticity $\bar{d}\frac{\rho\alpha\eta}{1-\eta}$ that I target, although the two are close (Section 5.2). The exact lower bound on $\bar{d}$ is therefore potentially lower. The bound on ρ is unaffected, since $\beta_{IV} < \frac{\rho\alpha\eta}{1-\eta}$ holds exactly.

So long as $\bar{d} \in (0,1)$, the firm-size distribution has a gap, as in Figure 6, whose location is set by the adoption rate and whose size is set by ρ: by Proposition 4, the difference between the tail indices of firm size with and without adoption is $\zeta\rho\alpha/(1+\omega)$. Because at least 29% of firms adopt, the gap lies in the middle of the firm-size distribution rather than in its tail. Therefore, the middle is most informative about ρ. In addition to β_{IV}, I target the US firm-size distribution from the 10th to the 99.9th percentile, measuring size by employment in the BDS (Table 6b).

Scope Costs. By Proposition 4, the ratio of the tail indices of scope and firm size is $1+\omega$. To capture this ratio, I include as targets the 99th, 99.5th, and 99.9th percentiles of n_f. Given the tail of the firm-size distribution, targeted above, they are most informative about ω. From the first-order condition for n_f, the mean and variance of scope discipline the mean and variance of $\log f_{c,f}$, so I also target $\mathbb{E}(n_f)$ and $\mathbb{V}\text{ar}(\log n_f)$.

Technology. The tail of the firm-size distribution, targeted above, is most informative about the Pareto shape ζ. Core competency, δ, is informed by a regression of log product revenue on the product's log rank within the firm, with firm-year fixed effects, following Arkolakis et al. (2021). In the model, a firm's slope is $-\delta\frac{\eta}{1-\eta(1-\rho\alpha d_f)}$, so the estimated slope carries the information about δ given the other parameters.

Table 5: Parameters

Assigned			*Calibrated*		
Param.	Description	Value	Param.	Description	Value
σ	Elast. of substitution	3.00	*Scope economies*		
η	$(\sigma-1)/\sigma$	0.67	ρ	Non-rivalry	0.818
α	Mgmt. input share	0.35	f_e	Adoption cost	0.005
γ	Materials share	0.60	*Scope costs*		
ϕ	Org. capital share of SG&A	0.30	ω	Scope-cost curvature	1.250
			μ_c	Mean of $\log f_{c,f}$	-2.020
			σ_c	S.d. of $\log f_{c,f}$	0.804
			Technology		
			ζ	Pareto shape of z_f	3.584
			δ	Core competency	0.338

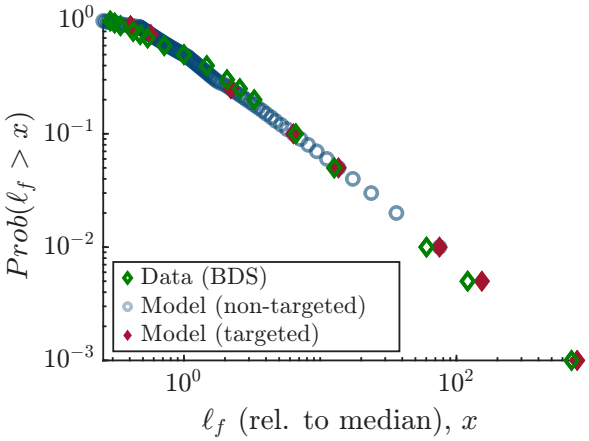
Model Fit. The adoption rate of non-rival management is 35%. The strength of non-rivalry is $\rho = 0.818$, close to full non-rivalry, because the adoption rate is near its floor of 29%. The calibrated model matches the targeted moments well. Among the moments it does not target, reported in the bottom rows of Table 6a, the model matches the sales share of the top 1% of firms closely and overstates the 90th and 95th percentiles of scope by about one product. Although the model targets only the IV estimate of the relationship between scale and scope, it also closely matches the raw relationship in the US data. See Figure B1.

Table 6: Model Fit

(a) Moments

Moment	Data	Model	Informs
Scope distribution			
n_{p99}	14.00	14.78	ω
$n_{p99.5}$	21.00	20.12	ω
$n_{p99.9}$	41.00	40.16	ω
$\mathbb{E}(n_f)$	2.16	2.32	μ_c
$\mathbb{V}\text{ar}(\log n_f)$	0.67	0.79	σ_c
Firm-size distribution			
Employment, p10–p99.9	Panel (b)		ρ, ζ
Regression coefficients			
β_{IV}	0.20	0.20	f_e
β_{core}	−0.59	−0.59	δ
Untargeted			
n_{p90}	4.00	4.83	–
n_{p95}	6.00	7.12	–
Top 1% sales share	0.70	0.71	–

(b) Firm-size distribution



Notes: US data. Scope moments are from NielsenIQ, with products defined as CMIE categories (see Section 2.1); firm size is employment from the BDS. *Informs* gives the parameter each targeted moment is most informative about (Section 5.3). β_{IV} is the IV estimate in Table 4; β_{core} is the slope of log product revenue on log product rank, with firm-year fixed effects. n_{p90}, n_{p95}, n_{p99}, $n_{p99.5}$, and $n_{p99.9}$ are percentiles of the number of products. Panel (b): complementary CDF of employment relative to the median.

6. Macroeconomic Implications of Distortions

In this section, I introduce size-dependent distortions and study their implications for aggregate output. I show how distortions lower aggregate productivity more if scope economies are present. Let $\tau(z_f) = \tau_0 z_f^{-\xi}$ be a firm-level revenue wedge.[29] Two parameters govern distortions: τ_0 sets the level of wedges and ξ controls the elasticity of wedges with respect to core productivity (and hence size). A wedge $\tau < 1$ acts like a tax. With $\xi > 0$, firms with higher z_f face a lower τ and are taxed more. Following the literature, I assume that the degree to which firms are distorted is a function of their core productivity. I write the firm's problem with $\tau(z_f)$ distorting total revenues:

$$\Pi_f^d = \max_{\substack{n_f, m_f, \{\ell_{i,f}, x_{i,f}, m_{i,f}\}_i \\ \{y_{i,f}, p_{i,f}\}_i}} \tau(z_f) \int_0^{n_f} p_{i,f} y_{i,f}\, di - W \int_0^{n_f} \ell_{i,f}\, di - P \int_0^{n_f} x_{i,f}\, di - P_m^d m_f - \mathcal{C}_f(n_f). \tag{35}$$

[29] Revenue from the wedges is rebated lump-sum to households.

Since these distortions are a function only of z_f and not of firm choices, they act as a shifter of firm productivity. As a result, profits in Proposition 1 become

$$\Pi_f^d = \max_{n_f,\ell_f,x_f,m_f} A\left(\tilde{Z}_f \ell_f^{1-\alpha-\gamma} x_f^{\gamma} m_f^{\alpha}\right)^{\eta} - W\ell_f - Px_f - P_m^d m_f - \mathcal{C}_f(n_f) \tag{36}$$
$$\text{where} \quad \tilde{Z}_f = \tau(z_f)^{1/\eta} Z_f.$$

Effectively, I replace Z_f with $\tilde{Z}_f$ as the firm-level productivity. With product-level distortions, I also need to account for within-firm distortions. See Appendix B.4.1 for the derivations of this case.

6.1. Aggregation

In this subsection, I discuss aggregation in this economy and define aggregate productivity. To aid exposition, I assume $\phi = 0$. Hence, adoption confers only cost savings and no cost advantage, in the terms of Section 4.2.

Aggregate output can be expressed as

$$Y \propto Z^{\frac{1}{1-\gamma}} \bar{\tau}^{\frac{\gamma}{1-\gamma}}, \qquad Z = \left(\int \left(\frac{\tau(z_f)}{\bar{\tau}}\right)^{\frac{1}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} \, df\right)^{\frac{1-\eta}{\eta}}, \qquad \frac{1}{\bar{\tau}} = \int \frac{1}{\tau(z_f)} \frac{\tilde{Z}_f^{\frac{\eta}{1-\eta}}}{\int \tilde{Z}_{f'}^{\frac{\eta}{1-\eta}} \, df'} \, df. \tag{37}$$

Output, Y, is a function of aggregate productivity Z and the factor-share wedge $\bar{\tau}$. First, Z aggregates across firms similarly to how a firm without scope economies aggregates across products. Second, $\bar{\tau}$ is a harmonic sales-weighted average of firm-level distortions. To see this, note that the factor share of income can be written as $\eta\bar{\tau}$. Since η is the inverse markup due to the CES technology of the final-good producer, $\bar{\tau}$ reflects deviations from this benchmark due to the firm-level distortions. Hence, the aggregate markup in the economy is $\frac{1}{\eta\bar{\tau}}$. Since $\tau < 1$ acts like a tax, a lower $\bar{\tau}$ lowers the factor share and raises the aggregate markup. Appendix B.4.2 derives these expressions, Appendix B.4.4 extends them to product-level distortions, and Appendix B.4.3 restates the results with $\phi > 0$, allowing a cost advantage in adoption.[30]

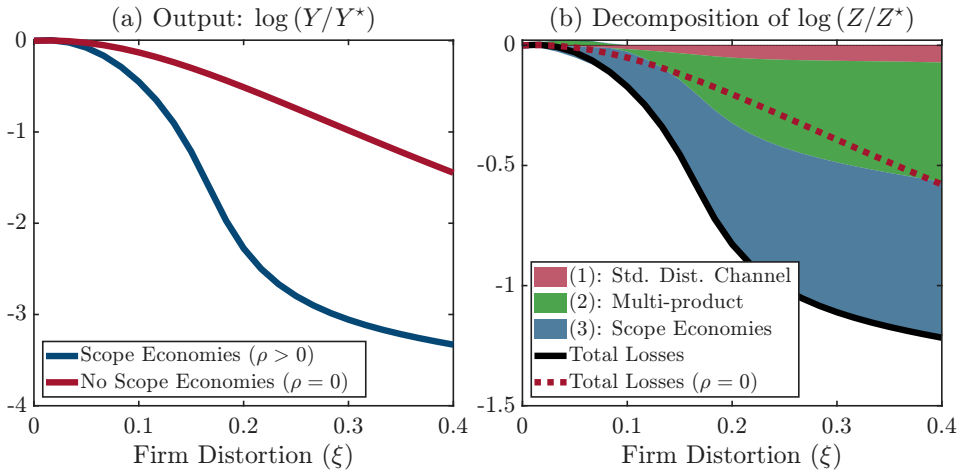
6.2. Misallocation

To isolate the role of misallocation, I consider an exercise that varies ξ while adjusting τ_0 so that $\bar{\tau} = 1$. That is, I increase the strength of the distortions but subsidize firms via τ_0 enough to keep the aggregate markup at $1/\eta$. Because $Y \propto Z^{\frac{1}{1-\gamma}} \bar{\tau}^{\frac{\gamma}{1-\gamma}}$ in equation (37), fixing $\bar{\tau} = 1$ means that distortions affect output only through Z. Panel (a) of Figure 7 plots the log difference of output relative to the undistorted economy, in which $\tau(z_f) = 1$ for all firms, and

[30] I show that when $\phi = 0$, so that adoption confers no cost advantage, aggregate productivity coincides with the Solow residual of a representative firm's production function. However, when $\phi > 0$, so that adoption can confer a cost advantage, the Solow residual conflates changes in the cost share of intermediates with productivity.

panel (b) decomposes the corresponding loss in aggregate productivity. I denote the undistorted values with a superscript $\star$. To illustrate amplification due to scope economies, I compare the outcomes to a model with $\rho = 0$, in which I recalibrate the remaining parameters to match the same moments.[31]

Figure 7: Implications of Size-Dependent Distortions



Output Losses. Panel (a) of Figure 7 shows that aggregate output falls by substantially more, in log points, with $\rho > 0$ than with $\rho = 0$. At $\xi = 0.2$, the loss in Z is about four times as large with $\rho > 0$ as with $\rho = 0$. Since distortions affect output only through Z, it suffices to decompose $\log(Z/Z^\star)$.

Decomposing Aggregate Productivity. To understand why distortions have larger effects in an economy with scope economies than in one without, consider the decomposition

$$\log\frac{Z}{Z^\star} = \frac{1-\eta}{\eta}\Bigg(\underbrace{\log\mathbb{E}\Big[\Big(\frac{\tau(z_f)}{\bar{\tau}}\Big)^{\frac{1}{1-\eta}}\Big] + \log\Lambda}_{\substack{\text{(1) Standard}\\ \text{Distortionary Channel}}} + \underbrace{\log\frac{\mathbb{E}\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big]}{\mathbb{E}\big[(Z_f^{0\star})^{\frac{\eta}{1-\eta}}\big]}}_{\text{(2) Multi-product}} + \underbrace{\log\frac{\mathbb{E}\big[Z_f^{\frac{\eta}{1-\eta}}\big]/\mathbb{E}\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big]}{\mathbb{E}\big[(Z_f^\star)^{\frac{\eta}{1-\eta}}\big]/\mathbb{E}\big[(Z_f^{0\star})^{\frac{\eta}{1-\eta}}\big]}}_{\substack{\text{(3) Scope}\\ \text{Economies}}}\Bigg), \tag{38}$$

where

$$\Lambda \equiv 1 + \frac{\mathrm{Cov}\left(\tau(z_f)^{\frac{1}{1-\eta}},\, Z_f^{\frac{\eta}{1-\eta}}\right)}{\mathbb{E}\big[\tau(z_f)^{\frac{1}{1-\eta}}\big]\,\mathbb{E}\big[Z_f^{\frac{\eta}{1-\eta}}\big]}$$

and Z_f^0 is the productivity of firm f at its optimally chosen scope but with $d_f = 0$. This decomposition splits $\log(Z/Z^\star)$ into three components. I refer to component (1) as the standard distortionary channel, since similar expressions arise in single-product models with size-dependent

[31] Table B1 contains this calibration's parameters.

distortions (Hopenhayn, 2014). Its first term is positive if firms are subsidized on average and negative if they are taxed on average. The second term, $\log \Lambda$, is the misallocation channel, as it reflects the covariance between wedges and productivity. It is negative if productive firms are taxed more heavily than less productive firms.

Since firm-level productivity depends on scope (n_f) and firm organization (d_f), components (2) and (3) separate the contributions of the two to aggregate productivity. I refer to component (2) as the multi-product channel. It compares average firm-level productivity in the two economies when no firm adopts non-rival management, so it moves only through scope. Component (3) is the scope-economies channel. It reflects the change in the productivity that firms gain by adopting non-rival management, relative to the undistorted economy. It equals zero when no firm adopts ($\bar{d} = 0$) or when $\rho = 0$, since then $Z_f = Z_f^0$ for every firm.

Contribution of Each Component. Panel (b) of Figure 7 stacks the three components of equation (38) as ξ varies, and the solid line is their sum. Component (3), the scope-economies channel, accounts for 65% of the losses. The share of the standard distortionary channel is 6% to 11% and falling with ξ. For ξ up to about 0.3, the losses in the $\rho = 0$ model, the dashed line in panel (b), closely track the sum of components (1) and (2). Therefore, the standard distortionary and multi-product channels summarize the losses in an economy without scope economies, and component (3) measures the additional losses due to accounting for scope economies.

Extensive and Intensive Margins. Component (3) moves along two margins: how many firms adopt non-rival management and how much adoption raises the productivity of adopters. Let $\mathbb{E}_1[\cdot] \equiv \mathbb{E}[\cdot \mid d_f = 1]$ denote the average over adopters. Then $\mathbb{E}\big[Z_f^{\frac{\eta}{1-\eta}}\big]/\mathbb{E}\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big] = 1 + \bar{d}\,\mathcal{R}\,\mathcal{G}$, where

$$\mathcal{R} \equiv \frac{\mathbb{E}_1\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big]}{\mathbb{E}\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big]}, \qquad \mathcal{G} \equiv \frac{\mathbb{E}_1\big[Z_f^{\frac{\eta}{1-\eta}}\big]}{\mathbb{E}_1\big[(Z_f^0)^{\frac{\eta}{1-\eta}}\big]} - 1.$$

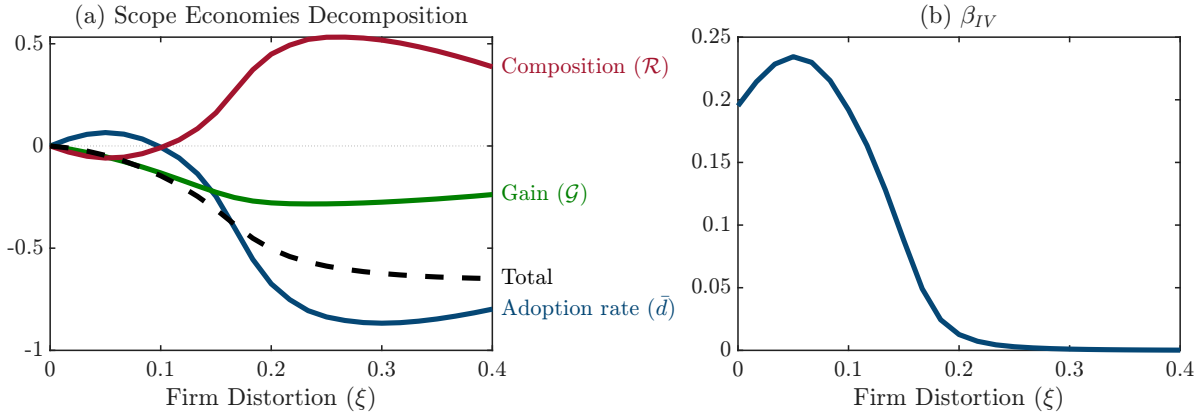
Component (3) then decomposes into three pieces,

$$(3) = \frac{1-\eta}{\eta}\,\bar{\mathcal{W}}\Bigg\{ \underbrace{\log \frac{\bar{d}}{\bar{d}^\star}}_{\text{Extensive Margin}} + \underbrace{\underbrace{\log \frac{\mathcal{R}}{\mathcal{R}^\star}}_{\text{Composition}} + \underbrace{\log \frac{\mathcal{G}}{\mathcal{G}^\star}}_{\text{Gain from Adoption}}}_{\text{Intensive Margin}} \Bigg\}, \tag{39}$$

where $\bar{\mathcal{W}}$ is a weight, defined in Appendix B.4.2. The extensive margin is the change in the adoption rate, $\bar{d}$. The intensive margin has two terms. The first, $\mathcal{R}$, reflects the composition of adopters, measured by their size relative to that of all firms, with both evaluated without non-rival management. The second, $\mathcal{G}$, is the gain from adoption, the proportional increase in productivity that adopters obtain from non-rival management. The extensive margin and composition move in opposite directions when the adoption threshold changes. If distortions

raise the threshold, the firms that stop adopting are the smallest adopters, so $\bar{d}$ falls and $\mathcal{R}$ rises.

Figure 8: Decomposition of the Scope-Economies Channel



Panel (a) of Figure 8 plots the three terms in equation (39) and the sum. For small distortions, the adoption rate rises slightly relative to the undistorted economy, which reflects the subsidy τ_0 that normalizes $\bar{\tau}$. For low ξ, losses are due to the intensive margin, as both composition and the gain from adoption fall. As ξ increases further, distortions substantially discourage non-rival management, primarily through the extensive margin. The extensive margin matters because adoption is discrete. Each firm that distortions push below the threshold gives up a discrete jump in productivity. Between $\xi = 0.1$ and $\xi = 0.2$, the adoption rate falls the most. It is in this region that losses increase significantly and amplification is most pronounced.

Measured Scope Economies. Panel (b) of Figure 8 plots β_{IV} in the model. Because β_{IV} scales with $\bar{d}$, it follows the adoption rate in panel (a). It is close to the estimate of 0.20 for small distortions and falls toward zero as ξ rises. Therefore, large distortions can rationalize the absence of scope economies in India documented in Section 2.3, even though the technology that generates them is available. Next, I trace why distortions discourage adoption and how this amplifies the losses.

6.3. Amplification

In this subsection, I explain why scope economies amplify the losses documented in the preceding subsection. I use the simplified model introduced in Section 4.3 to trace the margins behind this amplification.

Distortions and Adoption. In panel (a) of Figure 8, the adoption rate of non-rival management, $\bar{d}$, falls as misallocation increases. To understand why $\bar{d}$ falls as ξ increases, consider

the semi-elasticity of the adoption rate with respect to ξ, where τ_0 adjusts to maintain $\bar{\tau} = 1$:

$$\frac{d \log \bar{d}}{d\xi} = \frac{\zeta}{\eta - \xi} \left\{ \frac{d \log \tau_0}{d\xi} + \left(\frac{1-\eta}{1-\gamma} - \eta \right) \frac{d \log Z}{d\xi} - \log \bar{z} \right\}. \tag{40}$$

The semi-elasticity has three terms, each scaled by $\frac{\zeta}{\eta - \xi}$, which is positive since $\xi < \eta$. The first term is the derivative of $\log \tau_0$ with respect to ξ. It reflects how much the subsidy τ_0 must rise to eliminate the aggregate markup wedge. The second term is the response of Z to the distortion. In Figure 7, Z falls as ξ increases. If $\frac{1}{1-\gamma} > \frac{\eta}{1-\eta}$, then, all else equal, a fall in aggregate productivity leads to a further fall in the adoption rate. If this inequality is reversed, a fall in Z raises the adoption rate. The final term is the direct effect of increasing ξ, where $\bar{z}$ is the threshold productivity for adoption. Since ξ enters the marginal adopter's wedge as an exponent, this effect is proportional to $\log \bar{z}$. It reflects how a higher ξ raises the threshold $\bar{z}$ by distorting the revenues of the marginal adopter.

This feature of the model, that a fall in Z lowers adoption, is shared with Buera et al. (2026). In the exercise of Figure 7, the feedback through Z is strong enough that the adoption rate falls as ξ increases, even though τ_0 rises. This is due to the high materials share, γ. As noted in Buera et al. (2026), when distortions become more severe, the wage falls and the price of materials relative to the wage rises, which makes adopting non-rival management less attractive.[32]

Distortions and Firm-Level Productivity. How does varying ξ change firm-level productivity Z_f? First, consider the case of a non-adopting firm:

$$\frac{d \log Z_f}{d\xi}\Big|_{d_f=0} = \frac{1}{\eta} \frac{1}{\omega} \left(\frac{d \log \tau_0}{d\xi} + \left(\frac{1-\eta}{1-\gamma} - \eta \right) \frac{d \log Z}{d\xi} - \log z_f \right). \tag{41}$$

Equation (41) is the semi-elasticity of Z_f to the size distortion ξ for a firm that does not adopt non-rival management. As in equation (40), the first two terms reflect the effect of τ_0, the common wedge adjusting to maintain $\bar{\tau} = 1$, and feedback effects from aggregate productivity Z. The latter is similarly important because a fall in Z reduces the market size, which affects the cost of expanding scope and hence firm-level productivity. Note that, as with equation (40), if $\frac{1}{1-\gamma} > \frac{\eta}{1-\eta}$, there is amplification due to intermediate materials in production. The final term is the direct effect. Since ξ enters as an exponent, this effect is proportional to the firm's own core productivity, $\log z_f$.

For adopters, the semi-elasticity of Z_f to ξ is

$$\frac{d \log Z_f}{d\xi}\Big|_{d_f=1} = \frac{\omega(1+g)}{\omega - g} \frac{d \log Z_f}{d\xi}\Big|_{d_f=0}, \tag{42}$$

where g is, as in Section 4.3, the elasticity of revenue per product with respect to the number

[32] If $\gamma = 0$, a fall in Z lowers adoption only if $\eta < 1/2$.

of products among adopters. The adopter's semi-elasticity is that of the non-adopter scaled by $\frac{\omega(1+g)}{\omega-g}$. First, this term is greater than or equal to one, since $\omega > g$. Second, g is increasing in ρ, so stronger non-rivalry leads to stronger amplification. The next step is to trace this firm-level amplification to macroeconomic outcomes.

Distortions and Aggregate Productivity. The following proposition uses equations (40), (41), and (42) to characterize the semi-elasticity of aggregate productivity Z to ξ, the degree of size dependence in $\tau(z_f)$.

Proposition 6. *In the simplified model with $\phi = 0$, suppose that τ_0 adjusts to hold $\bar{\tau} = 1$ as ξ varies. In the undistorted economy, $\xi = 0$, the semi-elasticity of aggregate productivity to ξ is*

$$\frac{d\log Z}{d\xi} = \frac{1}{1-\mathcal{M}}\Bigg(\underbrace{\bar{d}^S \mathbb{E}_1^S\left[\frac{\partial \log\left(Z_f^1/Z_f^0\right)}{\partial \xi}\right]}_{\mathcal{I}\text{: intensive margin}} + \underbrace{\Delta^S\,\bar{d}\,\frac{\partial \log \bar{d}}{\partial \xi}}_{\mathcal{E}\text{: extensive margin}}\Bigg), \tag{43}$$

where $\mathcal{M}$ is the roundabout multiplier,

$$\mathcal{M} = \frac{1-\eta}{\eta}\frac{1}{\omega}\left(\frac{1}{1-\gamma} - \frac{\eta}{1-\eta}\right)\left(1 + \bar{d}^S\,\frac{g(1+\omega)}{\omega - g} + \bar{d}\,\omega\zeta\,\Delta^S\right).$$

Here $\Delta^S \equiv \frac{1-\eta}{\eta}\frac{\bar{S}^1 - \bar{S}^0}{\mathbb{E}[S_f]}$ is the jump in the sales of the marginal adopter, and $\bar{S}^0$ and $\bar{S}^1$ are the sales of a firm at the threshold without and with adoption. The partial $\partial/\partial\xi$ is the direct effect of ξ, holding Z fixed, with τ_0 adjusting to hold $\bar{\tau} = 1$:

$$\frac{\partial \log\left(Z_f^1/Z_f^0\right)}{\partial \xi} = -\frac{1}{\eta}\,\frac{1+\omega}{\omega}\,\frac{g}{\omega - g}\left(\log z_f - \mathbb{E}^S[\log z_f]\right), \qquad \frac{\partial \log \bar{d}}{\partial \xi} = \frac{\zeta}{\eta}\left(\mathbb{E}^S[\log z_f] - \log \bar{z}\right),$$

where $\mathbb{E}^S[\cdot]$ is the sales-weighted mean, $\mathbb{E}_d^S[\cdot] \equiv \mathbb{E}^S[\cdot \mid d_f = d]$, and $\bar{d}^S$ is the adopters' sales share.

The proof is in Appendix B.4.5. Proposition 6 decomposes the response into intensive and extensive margins of non-rival management, $\mathcal{I}$ and $\mathcal{E}$, scaled by $1/(1-\mathcal{M})$, where $\mathcal{M}$ is the roundabout multiplier. In this exercise, τ_0 adjusts to make the distortion revenue neutral, so if no firm adopts ($\bar{d} = 0$), size-dependent distortions have no first-order effect.

Intensive Margin. I now turn to the terms in Proposition 6, beginning with the intensive margin, $\mathcal{I}$. This margin captures how the distortions erode adopters' gains from adoption, holding adoption fixed. Here, $\frac{\partial \log\left(Z_f^1/Z_f^0\right)}{\partial \xi}$ is the direct effect of ξ on a firm's gain from adoption, $\log Z_f^1 - \log Z_f^0$. Substituting for the partial derivative,

$$\mathcal{I} = -\bar{d}^S(1-\bar{d}^S)\,\frac{1}{\eta}\,\frac{1+\omega}{\omega}\,\frac{g}{\omega - g}\left(\mathbb{E}_1^S[\log z_f] - \mathbb{E}_0^S[\log z_f]\right).$$

This expansion shows where the erosion comes from. Non-rival management makes firm-level productivity more responsive to z_f (Section 4.3), so the gains from adoption increase with z_f, and more so when g is larger. If $\rho = 0$, then $g = 0$ and $\mathcal{I} = 0$, since, without non-rivalry, adoption does not change how firm-level productivity responds to distortions. Because a distortion taxes high-z_f firms, it erodes the gains of the largest adopters most. The erosion is larger the wider the size gap between adopters and non-adopters, $\mathbb{E}_1^S[\log z_f] - \mathbb{E}_0^S[\log z_f]$, since adopters then bear more of the tax. Finally, the factor $\bar{d}^S(1 - \bar{d}^S)$ makes the intensive margin hump-shaped in $\bar{d}^S$, as $\mathcal{I} = 0$ if no firm or every firm uses non-rival management.

Extensive Margin. The extensive margin, $\mathcal{E}$, captures the gains forgone by marginal firms that no longer adopt. It is the product of the fraction of adopters, $\bar{d}$, the sales-weighted gain of the marginal adopter, Δ^S, and the semi-elasticity of the adoption rate to ξ. Substituting for the semi-elasticity, the extensive margin can be expressed as

$$\mathcal{E} = \bar{d}\Delta^S \frac{\zeta}{\eta} \left(\mathbb{E}^S[\log z_f] - \log \bar{z} \right).$$

Whether the extensive margin attenuates or amplifies losses depends on the size of the marginal adopter relative to the sales-weighted average firm. This is because, as ξ increases, τ_0 rises to hold the aggregate wedge at $\bar{\tau} = 1$. Firms with core productivity below the sales-weighted mean, $\log z_f < \mathbb{E}^S[\log z_f]$, receive a net subsidy and firms above the mean a net tax. In partial equilibrium, if marginal adopters are small ($\log \bar{z} < \mathbb{E}^S[\log z_f]$), the subsidy encourages adoption and $\mathcal{E}$ attenuates losses; if they are large, the tax discourages adoption and $\mathcal{E}$ amplifies losses.

Roundabout. The final term to discuss is the roundabout multiplier, $\mathcal{M}$, which reflects how changes in Z feed back to affect firms' decisions. This feedback is a source of strategic complementarity that is important for amplification. What is unique to this model is that roundabout amplification contains additional feedback due to scope economies. To see this, consider the following rewriting of $\mathcal{M}$:

$$\mathcal{M} = \underbrace{\frac{\partial \log Z_f^0}{\partial \log Z}}_{\mathcal{M}|_{\rho=0}} + \underbrace{\bar{d}^S \frac{\partial \log \left(Z_f^1 / Z_f^0 \right)}{\partial \log Z}}_{\text{intensive}} + \underbrace{\Delta^S \bar{d} \frac{\partial \log \bar{d}}{\partial \log Z}}_{\text{extensive}}. \tag{44}$$

Equation (44) expresses $\mathcal{M}$ in terms of the partial derivatives of firm-level productivity and of the adoption rate.[33] If there are no scope economies, then $\mathcal{M} = \frac{\partial \log Z_f^0}{\partial \log Z}$, which is proportional to $\left(\frac{1}{1-\gamma} - \frac{\eta}{1-\eta} \right)$, the familiar multiplier due to roundabout production. However, as discussed after equation (42), scope economies mean that firms are also more responsive to aggregate variables. Hence, in addition to the usual roundabout term, if enough firms adopt and non-

[33] Note that the responses of firms' Z_f in the simplified model to changes in Z are homogeneous conditional on the adoption choice d_f.

rivalry is strong (high ρ), there is an additional *intensive* margin that increases amplification through Z. This term is positive when $\frac{1}{1-\gamma} > \frac{\eta}{1-\eta}$ and adopters' Z_f rises more with Z than non-adopters' Z_f. There is also an *extensive* margin, as a higher Z raises the adoption rate, $\bar{d}$ (equation (40)), and each new adopter contributes the sales-weighted productivity jump, Δ^S. These margins mirror those in Proposition 6.

Connection to the Data. Wherever non-rival management enters Proposition 6, amplification due to scope economies runs through g, the elasticity of revenue per product with respect to the number of products among adopters (Section 4.3). It governs the erosion of adopters' productivity gains in $\mathcal{I}$, the jump in sales upon adoption (Δ^S) in $\mathcal{E}$, and additional feedback through roundabout production in $\mathcal{M}$. All three vanish if $g = 0$, leaving only the standard roundabout term in $\mathcal{M}$. At the calibrated $g \approx 0.57$, adopters' firm-level productivity responds to ξ about $\frac{\omega(1+g)}{\omega-g} \approx 2.90$ times as much as non-adopters'. Unlike in models of technology adoption, where gains from adoption are inferred indirectly from other margins, g is disciplined directly by the elasticity of revenue per product with respect to scope (Section 5.3).

7. Quantifying Distortions in India

In this section, I conduct two exercises to take the model to India. First, in the partial recalibration, I hold parameters fixed at their US values and introduce distortions to match India's firm-size distribution and the absence of scope economies. This exercise isolates the source of amplification. Second, in the full recalibration, my main exercise, I also discipline the model with India's scope distribution. I then show why these results require a sufficiently large materials share γ and consider a robustness check.

Recall two important differences between India and the US. First, the firm-size distribution is much thicker-tailed in the US (Figure 1a). In the BDS data used to calibrate the US firm-size distribution, Kondo et al. (2023) estimate a tail index of 1.10. For India, the estimated tail index for employment from the 2005 Economic Census is 1.64.[34] Second, revenue per product rises with the number of products in the US but not in India (Section 2.3). Since the OLS bias is upward, the flat slope in India cannot reflect bias masking scope economies.[35]

7.1. Identifying Distortions

I add ξ and τ_0 to the US calibration and choose their values to match these differences. First, note that ξ thins the tail. Proposition 4 becomes

$$\zeta_\ell^{(d)}(\xi) = \frac{\eta}{\eta - \xi}\,\zeta_\ell^{(d)}, \qquad d \in \{0, 1\}, \tag{45}$$

[34] From the 2007 ASI for manufacturing establishments, it is 1.73. Both estimated tail indices for India are computed via the method in Gabaix and Ibragimov (2011).

[35] See columns (4)–(6) of Table A6.

where $\zeta_\ell^{(d)} = \zeta_n^{(d)}/(1+\omega)$ is the tail index of the labor distribution. I assume that $\xi < \eta$ throughout to avoid reversing firm ranks. Therefore, ξ can raise the tail index in the US to that of India. For the second difference, scope economies, note that the common distortion, τ_0, also plays a role. A lower τ_0 lowers $\bar{d}$, the adoption rate,[36] and hence the elasticity of revenue per product with respect to the number of products, $\beta_{IV} \approx \bar{d}\frac{\rho\alpha\eta}{1-\eta}$. Because only the largest firms adopt, lowering adoption also thins the tail of the firm-size distribution (Section 4.4), which is the role ξ plays. In this sense, the common distortion acts like a size-dependent one. Hence, ξ is disciplined by the relative tail indices of the US and India, and τ_0 by the absence of scope economies in India.[37]

7.2. Holding US Parameters Fixed

To calibrate ξ, I target the 99th, 99.5th, and 99.9th percentiles of labor relative to the median in India, using the 2005 Economic Census. To calibrate τ_0, I target $\beta_{IV} = 0.01$, which is conservative about the gap in scope economies between the US and India, implying an adoption rate of 2%. Panel A of Table 7 reports the results of this calibration relative to the US, along with the results of applying the same ξ and τ_0 to the calibration without scope economies (Table B1). In panel B, I repeat this exercise but assume that India and the US have the same scope economies.

Table 7: Effect of Distortions with US Parameters Held Fixed

	ξ	τ_0	Y (US = 1)	Z (US = 1)	C (US = 1)
Panel A: Near-zero adoption in India (2%)					
With scope economies ($\rho > 0$)	0.18	1.15	0.10	0.44	0.11
Without scope economies ($\rho = 0$)	0.18	1.15	0.47	0.80	0.51
Panel B: US-level adoption in India (35%)					
With scope economies ($\rho > 0$)	0.32	1.50	0.40	0.64	0.29
Without scope economies ($\rho = 0$)	0.32	1.50	0.45	0.68	0.38

In panel A, there is substantial amplification of the effect of distortions on output, consumption, and productivity. Output in India is 10% of the US level, consumption is 11%, and productivity is 44%. The same distortions in the model without scope economies reduce output to 47%. For context, in 2005, GDP per worker in the US was about 12 times that in India.[38] The model with scope economies implies that US output is about 10 times India's and can rationalize about 83% of the gap in GDP per worker, while the model without scope economies implies about twice India's output and can rationalize only about 18%.

In panel B, by contrast, there is little amplification. With and without scope economies,

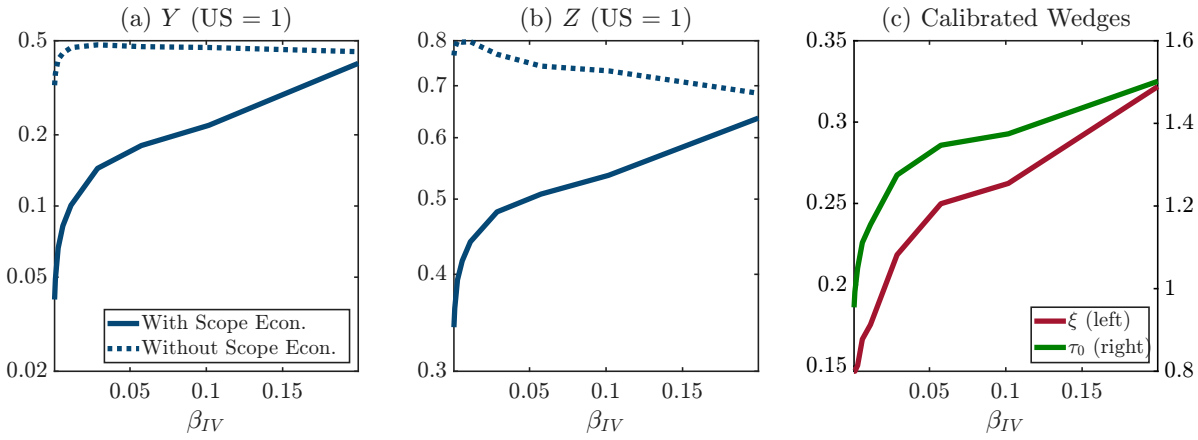
[36] See Appendix B.4.6 for the elasticity of $\bar{d}$ to τ_0 in the simplified model.

[37] Note that ξ also lowers the adoption rate, as reported in equation (40). However, it alone cannot match the tail index and the adoption rate together.

[38] See Penn World Table 10.0.

the models imply that the US has about 2.5 and 2.2 times the output of India, respectively. Distortions cause larger losses by eliminating scope economies. When they do not eliminate scope economies, size-dependent distortions play a limited role. Figure 9 presents this exercise for a range of intermediate counterfactuals, with adoption rates in India from 0.1% up to the US level. Panels (a) and (b) plot Y and Z against β_{IV}, compared with the $\rho = 0$ calibration. Panel (c) plots the calibrated values of ξ and τ_0, which I also apply to the $\rho = 0$ calibration.

Figure 9: Effect of Distortions Calibrated to India



The figure confirms that losses in the model with scope economies are greater when the adoption rate of non-rival management is low. At adoption rates below about 0.2%, $\tau_0 < 1$, so firms are taxed to generate such low adoption. As the adoption rate increases, scope economies in India approach those in the US, so the differences between the two countries are driven primarily by the tail index rather than by scope economies, and amplification shrinks.

Lastly, fitting the data requires weaker size dependence (lower ξ) and a larger common distortion (lower τ_0) as the targeted scope economies fall. Size dependence can be weaker because having fewer adopters already thins the tail (Section 4.4).

7.3. Full Recalibration

The partial recalibration holds all the parameters, other than ξ and τ_0, constant across India and the US. This does not guarantee a scope distribution consistent with India's, and it attributes the differences between India and the US entirely to distortions. In the full recalibration, I hold fixed only the technology parameters: the Pareto shape of core productivity z_f (ζ), the strength of non-rivalry (ρ), the adoption cost (f_e), and the curvature of the scope cost (ω). I allow core competency (δ) and the mean and standard deviation of $\log f_{c,f}$ (μ_c, σ_c) to vary, so the costs of maintaining scope can differ in India, reflecting differences in scope-related technology or in market-access costs. I calibrate these parameters, in addition to ξ and τ_0, jointly to the moments of the partial recalibration, together with India's scope moments and the within-firm

revenue–rank slope, β_{core}.[39] As before, I target $\beta_{IV} = 0.01$.

Table 8: Effect of Distortions in India: Full Recalibration

	(1) Baseline	(2) Baseline ($\bar{d}$ fixed)	(3) No Scope Econ ($\rho = 0$)
Panel A: Calibrated distortions			
ξ	0.112	0.112	0.216
τ_0	0.846	0.846	0.980
Panel B: Pre-reform level relative to post-reform			
Y (Reform = 1)	0.10	0.22	0.40
C (Reform = 1)	0.13	0.28	0.47
Panel C: Change from pre- to post-reform			
ΔAdoption (pp)	57.96	0.00	0.00
$\Delta\beta_{IV}$	0.25	0.00	—
$\%\Delta Z$	98.07	49.44	22.91

Table 8 reports the effects of a reform that removes distortions from the full recalibration, setting $\xi = 0$ and $\tau_0 = 1$. Unlike in Section 7.2, the US is not the undistorted counterpart of India, as scope costs and core competency differ across the two countries. The benchmark is instead India without distortions. I refer to the calibrated India economy as pre-reform and to the economy without distortions as post-reform. Column (1), the baseline, applies the reform to the full recalibration. Column (2) applies the same reform but raises f_e post-reform to hold the adoption rate $\bar{d}$ at its pre-reform value. Column (3) applies the reform in the model without scope economies, starting from the $\rho = 0$ US calibration (Table B1) and calibrating distortions analogously (Table B3). Since τ_0 is not identified in this model, I choose τ_0 such that the aggregate wedge, $\bar{\tau}$, is equal to that in the model with scope economies. Therefore, differences in losses across the two models are primarily attributable to aggregate productivity, Z.

Eliminating distortions has large effects in all columns. This partly reflects the large common distortion τ_0 and the high materials share γ, which amplifies the effect of distortions, as discussed in Proposition 6. In the baseline, pre-reform output is 10% of its post-reform level, compared with 40% without scope economies (column (3)). Relative to the gap in GDP per worker between the US and India, these output gains are about 80% and 20%, respectively. This is consistent with the results of the partial recalibration. Moreover, size dependence is weaker with scope economies ($\xi = 0.112$) than without them ($\xi = 0.216$), for the reasons discussed in Section 7.2.

In the baseline, productivity rises by about 100% from pre- to post-reform, compared with about 50% when adoption is held fixed (column (2)) and only 23% without scope economies (column (3)). Hence, scope economies amplify the effect of distortions on aggregate productivity along both the extensive and intensive margins, as predicted by Proposition 6. Most of this

[39] See Table B2 for the parameters and targeted moments.

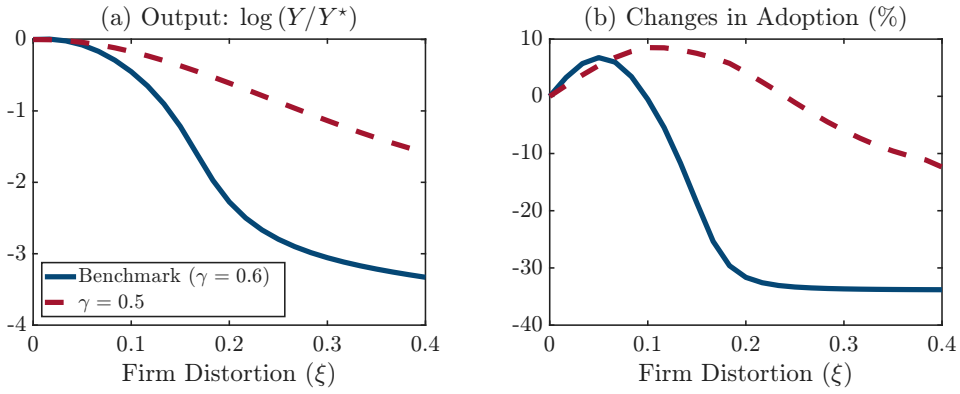
amplification comes from the extensive margin, as holding adoption fixed cuts the productivity gain in half. This is consistent with Section 6.2, where the scope-economies channel accounts for most of the losses (Figure 7) and, for large distortions, these losses come primarily from the extensive margin (Figure 8). Amplification through adoption is also central in Buera et al. (2026), where distortions lower the returns to adoption and fewer firms adopt.

An important feature of the baseline is that the reform raises adoption. The pre-reform adoption rate is about 58 percentage points below its post-reform level. Hence, the reform raises β_{IV} from 0.01 to about 0.26, close to the US value of 0.20.

7.4. The Role of the Materials Share

Recall from Section 6.3 that the adoption rate of non-rival management responds positively to Z, aggregate productivity, if $\frac{1}{1-\gamma} > \frac{\eta}{1-\eta}$, that is, if materials are a sufficiently large share of total costs. In the calibration, $\gamma = 0.6$ satisfies this condition, which, with $\eta = 2/3$, can be rewritten as $\gamma > 0.5$. At $\gamma = 0.5$, the feedback from Z to adoption vanishes. To study the implications, I recalibrate the model with $\gamma = 0.5$ and replicate the misallocation exercise of Section 6.2 in Figure 10. The response of adoption to ξ is now muted, and so is the response of output relative to the baseline with $\gamma = 0.6$. At $\xi = 0.4$, adoption is only about 12 percentage points lower than in the undistorted economy, compared with about 34 percentage points lower when $\gamma = 0.6$. With $\gamma = 0.5$, reaching zero adoption requires ξ in excess of η, where rank reversals occur. Hence, rationalizing the absence of scope economies in India requires a sufficiently large materials share.

Figure 10: Implications of Size-Dependent Distortions: Lower Materials Share



7.5. Robustness to the Common Distortion

The results in Section 7.2 depend on the identification of τ_0. A low τ_0 generates losses through $\bar{\tau}$ that may be independent of losses through Z. To isolate the losses through Z, instead of varying τ_0, I raise f_e to match $\beta_{IV} = 0.01$ and set τ_0 such that $\bar{\tau} = 1$. Table 9 presents the results. Output is 11% of the US level, compared with 10% in panel A of Table 7 and 63%

without scope economies.

Table 9: Effect of Distortions with $\bar{\tau} = 1$

	ξ	τ_0	Y (US = 1)	Z (US = 1)	C (US = 1)
Near-zero adoption in India ($f_e = 0.0633$, adoption 2%)					
With scope economies ($\rho > 0$)	0.19	1.20	0.11	0.45	0.12
Without scope economies ($\rho = 0$)	0.19	1.32	0.63	0.83	0.61

8. Conclusion

A widely held view is that size-dependent distortions explain income differences across countries. However, standard models find that they account for only a small share of these differences. I show that accounting for scope economies increases the importance of size-dependent distortions in explaining cross-country income gaps. Using product-level data, I document patterns consistent with scope economies in the US but not in India. I rationalize these patterns in a multi-product firm model in which firms can adopt a non-rival managerial input that is shared across products, generating scope economies.

Since the gains from adoption rise with scope, large firms adopt, and the right tail of the firm-size distribution thickens. Size-dependent distortions shrink these firms and lead them to abandon non-rival management. Eliminating distortions in the model calibrated to India raises aggregate productivity by about 100%, four times the gain in a model without scope economies. With scope economies, distortions can account for 80% of the gap in GDP per worker between the US and India, versus 20% without them.

In the model, the productivity gains from adoption are disciplined by firm-level data. I use input-cost shocks to identify the elasticity of revenue per product with respect to the number of products. Together with the firm-size distribution, this elasticity pins down the strength of non-rivalry and the adoption rate, which jointly govern the degree of amplification. Therefore, how firms organize production shapes the aggregate costs of distortions.

There are several avenues for future research. One is the role of falling costs of organization capital in the rise of firm scope and concentration. This is especially relevant given that AI could allow firms to codify their organizational practices to a greater degree (Eisfeldt et al., 2026). Another avenue is to study the role of product-portfolio diversification and scope economies in managing risk, in the direction of Bhandari et al. (2026).

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Data Appendix

A.1. Data Construction

NielsenIQ RMS. NielsenIQ RMS is provided by the Kilts Center for Marketing at the University of Chicago Booth School of Business. It collects point-of-sale records from grocery, drug, mass merchandiser, and convenience stores, with observations at the UPC-store-week level. UPCs are commonly known as barcodes. See Argente et al. (2024) for details on the coverage of the RMS data. For each product, I define total sales as the sum of sales across stores and weeks in that year. Prices are unit values: total sales in the year divided by total quantity sold, where quantities are measured in weight or volume. Since NielsenIQ anonymizes products that are private label, I exclude these products from the final panel. Firms are identified using the GS1 US Data Hub. GS1 is the sole issuer of UPCs, which firms license for a fee. UPCs typically carry a five- to ten-digit prefix identifying the firm. I use GS1 data mapping each firm prefix to its owner, which links firms to UPCs. For the panel aggregated at the CMIE classification, I sum total sales for all UPCs belonging to a CMIE product category, producing a firm×product×year panel.

Prowess. In Prowess, I use the Identity, Standalone Annual Financial Statements, and Products Produced/Traded tables. The Identity module provides firm code, name, and other characteristics. The financial statements provide firm-level balance sheet information, such as gross wages, physical capital, and total sales. The products module breaks down firm sales by product. Products are classified according to CMIE's own internal classification, which is similar to India's National Industrial Classification (NIC) and approximately corresponds to the 6-digit level of the Harmonized System (HS). The product data require two adjustments. First, firms' reported product descriptions are more detailed than the CMIE classification. However, these descriptions are not standardized. For instance, a firm may report "Tea-Loose" and "Tea-Bagged," while another tea producer does not distinguish by packaging. Hence, I work at the CMIE category level rather than at the level of the raw product descriptions, which leave the definition of products to firm discretion. Second, the data include services, which need to be dropped from the analysis. To do this, I use CMIE's crosswalk to 5-digit NIC codes and drop CMIE categories that are services, such as all types of financial services. I define total sales of a product as its annual sales reported in Prowess. Prices are unit values: total sales in the year divided by total quantity sold, where quantities are measured in weight or volume. Firm-level information is provided for all the products. I keep only firms for which balance sheet information is provided. Finally, I create a CPG indicator equal to one for CMIE product categories matched to a NielsenIQ product module in the crosswalk described above.

Crosswalk. NielsenIQ RMS is organized into a hierarchy: each firm's UPCs are grouped into product modules, which are further nested into product groups. For example, the product module "Coffee - Soluble" contains all the UPCs that are unflavored instant coffee products, nested within the product group "Coffee." Product modules correspond most closely to the CMIE classification. To construct the crosswalk between NielsenIQ product modules and CMIE product categories, I first collect the module and category names. For each product module, I compute the Jaccard distance between its name and each CMIE category name based on word bigrams, and keep the five closest categories as candidates. I then manually select the best match from the pooled candidates. Table A1 shows four examples of the resulting mapping.

Table A1: Examples of the NielsenIQ–CMIE Crosswalk

Module code	NielsenIQ product module	NielsenIQ product group
CMIE category: Instant coffee		
1464	Coffee - Soluble Flavored	Coffee
1465	Coffee - Soluble	Coffee
CMIE category: Synthetic detergents		
7003	Detergents - Packaged	Detergents
7008	Detergents - Light Duty	Detergents
7012	Detergents - Heavy Duty - Liquid	Detergents
7020	Automatic Dishwasher Compounds	Detergents
7041	Detergent Boosters	Laundry Supplies
CMIE category: Razors & razor blades		
8487	Razor Blades	Shaving Needs
8488	Razors - Non-Disposable	Shaving Needs
8556	Razors - Disposable	Shaving Needs
7783	Razor Trimmer Appliance and Accessory	Housewares, Appliances
CMIE category: Mineral water/Bottled water		
1487	Water - Bottled	Soft Drinks - Non-Carbonated

Notes: Each panel lists all NielsenIQ product modules mapped to one CMIE product category. Each module maps to exactly one CMIE category, so a CMIE category groups one or more modules. Modules can come from different NielsenIQ product groups, and from different departments: razor trimmers belong to General Merchandise, while the other razor modules belong to Health & Beauty Care. CMIE categories can also be finer than NielsenIQ product groups: the remaining modules in the product group "Coffee" (ground and whole bean coffee, liquid coffee, and coffee substitutes) map to the CMIE category "Coffee." Module names are NielsenIQ module descriptions, shown in title case.

NielsenIQ–Compustat Panel. No public crosswalk exists between NielsenIQ RMS and Compustat. To construct one, I follow Argente et al. (2018) and match firms on the GS1 licensee name in four passes. To standardize names, I convert all characters to lowercase, eliminate punctuation, and replace common terms with their abbreviations, for example, "inc." for "incorporated" and "ltd." for "limited." In the first pass, I match firms on these standardized names. In the second pass, I eliminate the abbreviations and common terms from firm names, and match the remaining firms again on the cleaned names. In the third pass, I use the WRDS Subsidiary Data, based on Exhibit 21 of firms' 10-K filings, to match the remaining firms to

their parent companies in Compustat, repeating the first two passes. In the final pass, I take the 50 largest unmatched firms by number of products and manually check online whether each firm is publicly listed. Because matching is based on current firm names, firms that changed names during the sample period may be missed. Table A2 reports summary statistics for the matched panel and its coverage of NielsenIQ.

Table A2: Summary Statistics: NielsenIQ–Compustat Panel

	NielsenIQ–Compustat	All NielsenIQ
Panel A: Firms		
Share of firms with $n_{f,t} > 1$ (%)	85	75
Sales share of firms with $n_{f,t} > 1$ (%)	>99	>99
Sales share of firms above 99th pct. of $n_{f,t}$ (%)	18	48
Panel B: Share of NielsenIQ		
Products (%)	17	—
Sales (%)	35	—
Panel C: Sample		
Years	2006–2019	2006–2019
Firms	629	63,953
Firm-year observations	4,325	456,530

Notes: Products are defined as UPCs. Panel A shares are computed for each year; the table reports the mean across years. The 99th percentile of $n_{f,t}$ is computed over the pooled firm-year sample within each column.

A.2. ERP Adoption Events

This appendix describes how I date ERP adoption from firms' SEC filings for the case study in Section 2.4.

Filings. The sample consists of all firms in the NielsenIQ–Compustat panel with NielsenIQ sales between 2006 and 2019. For each firm, I download from EDGAR every 10-K and 10-Q whose reporting period falls within the firm's sample years, and extract the section on controls and procedures: Item 9A of the 10-K, which covers the fourth fiscal quarter, and Part I, Item 4 of the 10-Q, which covers the first three. A filing whose section cannot be located is dropped rather than partially coded. This yields 17,667 sections from 603 firms. The remaining firms lack an EDGAR identifier, filed under a predecessor registrant during the sample years, or have no section that can be parsed.

Coding. Each section is read and coded by a large language model (LLM). Since firms repeat much of this section from one filing to the next, paragraphs that are identical across a firm's filings apart from dates are read once, together with the dates each filing gives. Events are identified by reading every paragraph rather than by keyword search. For each paragraph that reports a change in internal control, the LLM records the type of system, the stage of

implementation (started, went live, completed, ongoing, or planned), the business units and functions named, the date of the event, and a verbatim quote of at most 15 words, which a script checks against the filing text. The date is the one stated in the text, if any; otherwise, it is the quarter that the filing covers. Table A3 shows an example.

Systems. Two types of systems define adoption. An ERP system is an enterprise resource planning system, named as such or by its vendor (for example, SAP, Oracle, or JD Edwards), or an integrated system covering order taking, the supply chain, and finance. A shared-services move consolidates finance, accounting, human resources, IT, or other back-office processes into a single shared-services center, or outsources them to a third party. Like an ERP system, a shared-services center replaces each unit's own back office with one that serves every product line. Other systems, such as general ledger or payroll software, are recorded but do not define adoption. I also exclude customer-facing systems, such as CRM and e-commerce platforms, and centralized purchasing, since gains from the latter could reflect buyer power rather than non-rivalry.

Adoption Events. A firm's adoption event is its first go-live or completion of an ERP or shared-services system, and its adoption year is the calendar year of that quarter. Later go-lives at the same firm, such as rollouts to other regions, do not define new events. A firm is treated if its first go-live falls between 2006 and 2019 and it has at least one year of NielsenIQ sales before its adoption year. Control firms are those whose filings never mention an ERP or shared-services system. I drop firms whose first go-live occurs before 2006, whose first go-live is dated only to the year, or whose filings mention an ERP or shared-services system without a dated go-live. The estimation sample contains 86 adopting firms, with adoption years from 2007 to 2019, and 420 control firms.

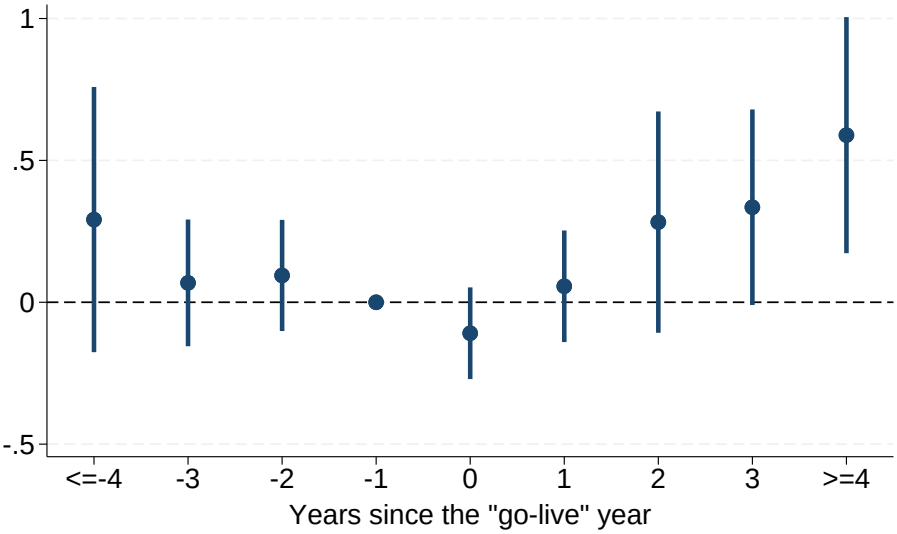
Table A3: Coding an ERP Adoption Event: Example

Filing	Text	Coded
Hasbro, 10-Q for the quarter ended June 27, 2010, Part I, Item 4	"During the **quarter ended June 27, 2010**, the **SAP upgrade was completed** . . ."	System: ERP (SAP upgrade) Stage: went live Units: U.S. and Canada Date: 2010 Q2

Notes: The text is verbatim from the filing; bold marks the words that date the adoption. Later filings report go-lives in Europe (2011) and Latin America (2013). The adoption event is the first go-live, so Hasbro's adoption year is 2010.

CMIE Products. Figure A1 repeats the event study in Figure 3 with products defined as CMIE categories, as in Section 2.1. As with UPCs, no coefficient is significant before adoption, and the coefficients increase after adoption.

Figure A1: Effect of ERP Adoption × Firm Scope on Revenue per Product: CMIE Products



Notes: Same as Figure 3, with products defined as CMIE categories and scope measured as the log number of CMIE products in the year before adoption.

A.3. Firm-Size Distribution

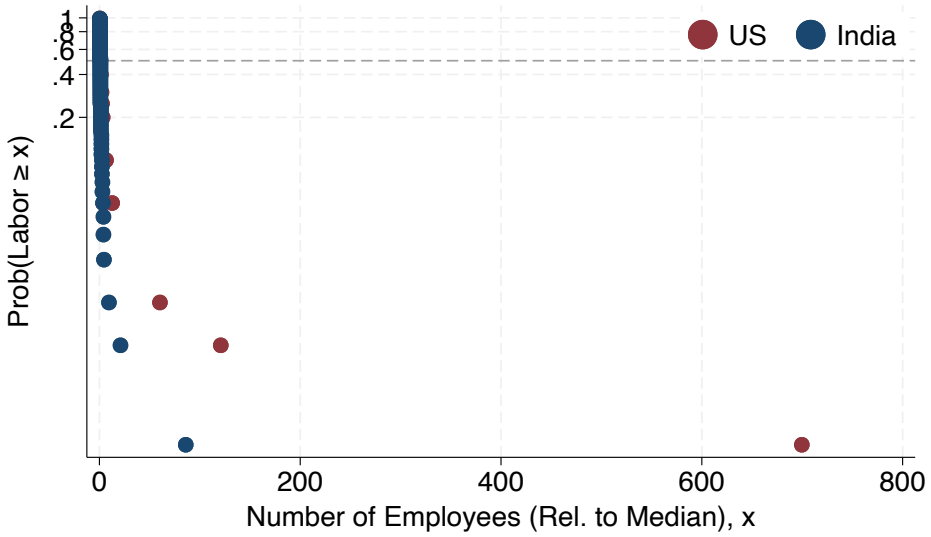
Firm size is the number of employees, measured using the 2005 Economic Census for India and 2007 Compustat for the US, without restricting to firms matched to NielsenIQ. This raises two concerns. First, the Economic Census enumerates establishments rather than firms. Since alternatives to Prowess also primarily survey establishments, I instead compare sales concentration in NielsenIQ and in Prowess: Table A4 reports the sales share of the top 25%, 10%, 5%, and 1% of firms. Second, Compustat omits smaller firms. Figure A2 instead compares India to the Census Bureau's Business Dynamics Statistics (BDS) for 2007. The BDS is excluded from the main figure only because its size bins yield a coarse complementary CDF.

Table A4: Sales Concentration: United States and India

	United States	India
Sales share of top firms (%)		
Top 25%	>99	93
Top 10%	97	84
Top 5%	92	75
Top 1%	70	58
Firms	29,642	3,182

Notes: Sales shares of firms in 2007. US: NielsenIQ RMS. India: Prowess.

Figure A2: Firm-Size Distribution: US (BDS) and India



Notes: Complementary CDF of firm employment, normalized by the median. US: Business Dynamics Statistics (BDS), 2007. India: 2005 Economic Census.

A.4. Robustness: Overview

Table A5 lists the robustness checks for the motivating fact in Section 2.3.

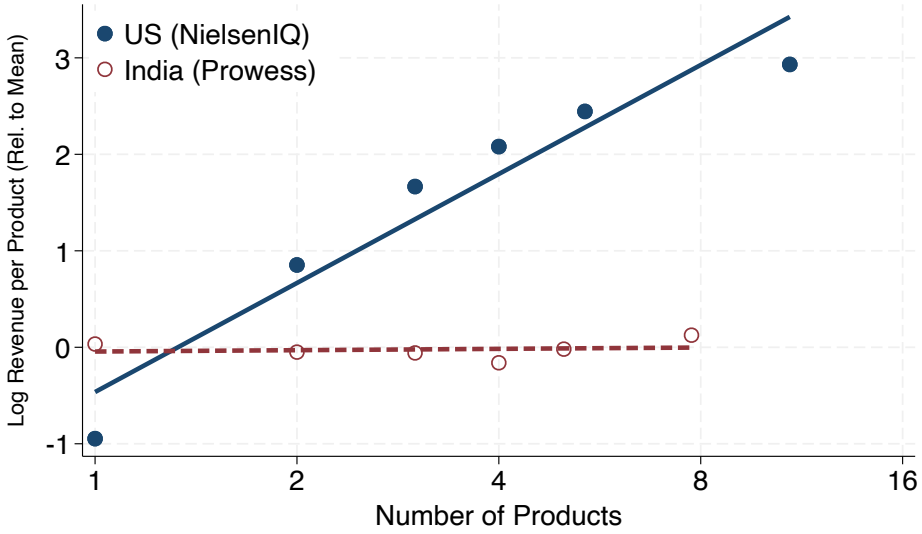
Table A5: Robustness Checks

Concern	United States	India
Panel A: Specification and measurement		
Visualization	Binned scatter (Fig. A3)	Binned scatter (Fig. A3)
Industry composition	Industry FE (Table A6)	Industry FE (Table A6)
Firm heterogeneity	Firm FE (Table A7)	—
Scope measure	UPCs (Fig. A6); inverse HHI (Fig. A4)	Inverse HHI (Fig. A4)
Scope and firm size	Compustat sales and assets (Fig. A8)	—
Partial-year sales	Continuing products (Fig. A5)	Continuing products (Fig. A5)
Sample period	—	2006–2019 (Table A8)
Panel B: Comparability		
Sectoral coverage	All CPG by construction	CPG products only (Fig. A9b)
Firm selection	Compustat firms (Fig. A7)	ASI establishments (Fig. A10)
Firm definition	Compustat parents (Fig. A7)	Business groups (Fig. A9a)

A.5. Binscatter: Revenue per Product and Number of Products

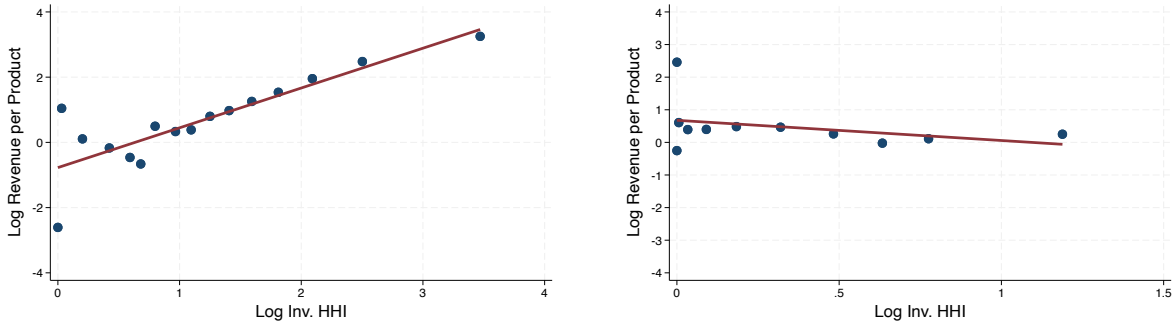
Figure A3 shows the relationship in Figure 2 as a binned scatter plot. Figure A4 measures scope as the inverse within-firm Herfindahl–Hirschman index of product sales shares. Because products in their first or last year are sold for only part of the year, Figure A5 computes revenue per product over continuing products only, while $n_{f,t}$ continues to count all of the firm's products. In all binned scatter plots, bins are quantiles of the variable on the horizontal axis; because many firms have the same number of products, bins are not of equal size.

Figure A3: Firm Scale and Scope, Binned Scatter: United States and India



Notes: Each point is the mean of log revenue per product within a bin of the number of products, relative to its mean in each country. The horizontal axis is on a log scale. Products are CMIE categories (see Section 2.1).

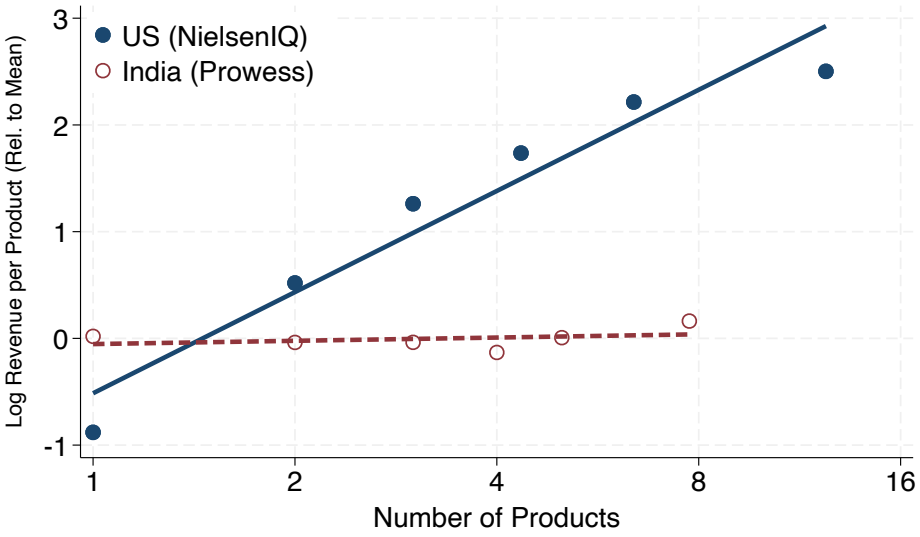
Figure A4: Firm Scale and Scope, Inverse HHI: United States and India



(a) United States (b) India

Notes: Binned scatter plots of log revenue per product, relative to its mean in each country, against the log inverse within-firm Herfindahl–Hirschman index of product sales shares. Single-product firms have a value of zero. Products are CMIE categories (see Section 2.1).

Figure A5: Firm Scale and Scope, Continuing Products: United States and India



Notes: Same as Figure A3, but revenue per product is averaged over continuing products only, that is, products that are not in their first or last observed year.

A.6. Elasticities: Revenue per Product and Number of Products

Tables A6 and A7 report the regression estimates discussed in Section 2.3, with log revenue per product as the dependent variable and the log number of products as the regressor. In Table A6, I include industry fixed effects. In Table A7, I include firm and age fixed effects. For the US, I use the NielsenIQ–Compustat panel, which provides the firm characteristics used as controls. In India, the elasticity is small and negative when controlling for age, and close to -1 with firm fixed effects. I do not interpret the latter as evidence of cannibalization. With firm fixed effects, the elasticity is identified only from changes in the number of products within firms. In Prowess, such changes are rare (Goldberg et al., 2010b) and may partly reflect reporting noise. Because the number of products appears on both sides of the regression, as the regressor and in the denominator of revenue per product, measurement error biases the elasticity toward -1. In NielsenIQ, by contrast, the number of products is measured from scanner records, consistent with the US estimate remaining positive.

Table A6: Revenue per Product and Scope: Industry Fixed Effects

	United States			India		
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Number of Products)	2.037***	1.931***	1.937***	0.031	-0.001	-0.001
	(0.135)	(0.122)	(0.131)	(0.028)	(0.026)	(0.026)
Year FE	Yes	Yes	–	Yes	Yes	–
Industry FE	No	Yes	–	No	Yes	–
Year × Industry FE	No	No	Yes	No	No	Yes
N	4500	4497	4236	116574	116573	116492
R^2	.292	.487	.516	.0249	.0955	.111

Notes: US: NielsenIQ–Compustat panel. Industry is 3-digit NAICS (US) and 3-digit NIC (India). Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A7: Revenue per Product and Scope: Age and Firm Fixed Effects

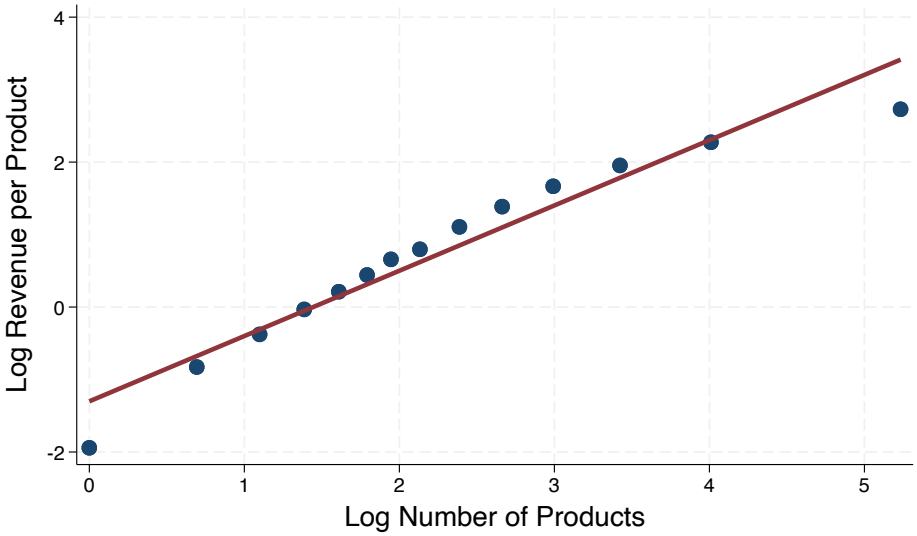
	United States			India		
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Number of Products)	1.930***	1.389***	1.313***	-0.071***	-0.814***	-0.827***
	(0.124)	(0.194)	(0.186)	(0.026)	(0.014)	(0.014)
Age FE	Yes	No	Yes	Yes	No	Yes
Firm FE	No	Yes	Yes	No	Yes	Yes
N	4485	4426	4415	116573	115710	115710
R^2	.492	.858	.861	.109	.894	.895

Notes: US: NielsenIQ–Compustat panel. All columns include year and 3-digit industry fixed effects. Age fixed effects are 15-year age bins. Firm age is measured from the date of incorporation: for the US, from Worldscope (Datastream) or, when unavailable, the first year the firm appears in Compustat; for India, from Prowess. Industry is 3-digit NAICS (US) and 3-digit NIC (India). Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

A.7. Sample: United States

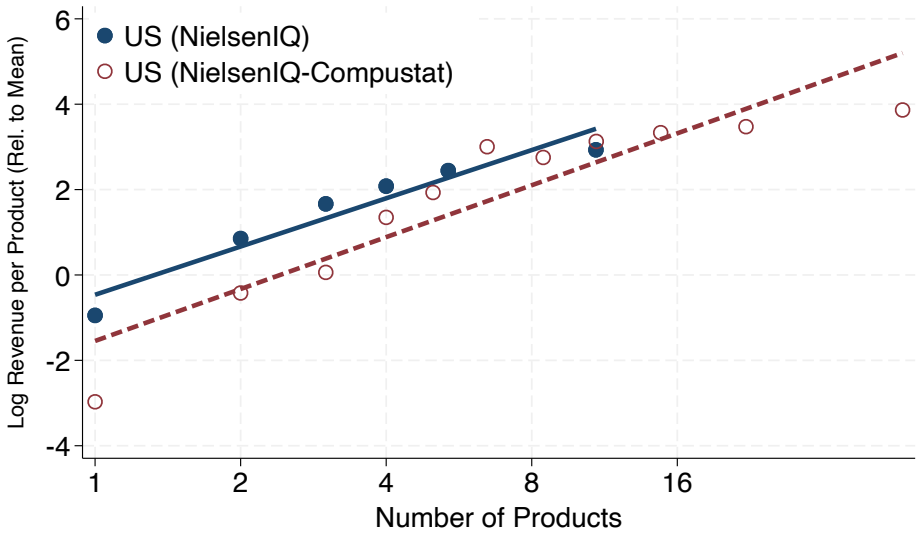
Figure A6 shows a binned scatter plot of log revenue per product on the log number of products, as in Figure A3, but defines scope as the number of UPCs. Figure A7 compares Figure A3 for the NielsenIQ panel and the one matched to Compustat. This addresses two concerns: firm selection relative to Prowess, and attributing sales and scope in NielsenIQ correctly to the parent company of the UPC licensee. Figure A8 plots the log number of products against log Compustat sales and total assets for the matched firms.

Figure A6: Firm Scale and Scope, UPCs: United States



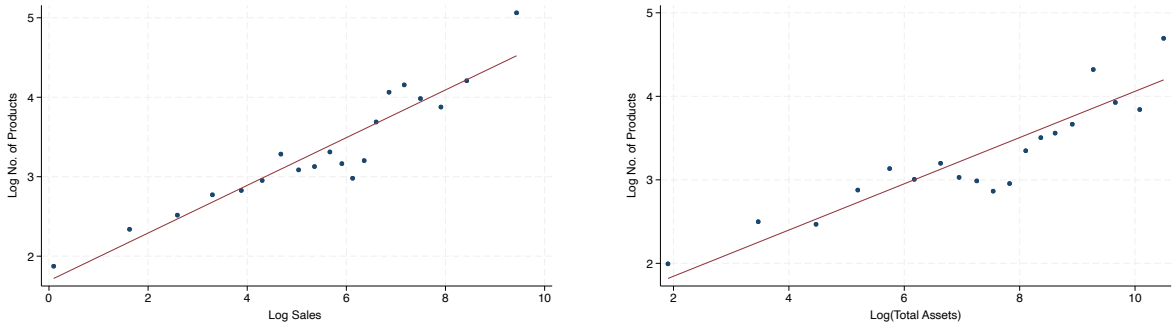
Notes: Binned scatter plot of log revenue per product, relative to its mean, against the log number of products for the US NielsenIQ RMS sample, with products defined as UPCs.

Figure A7: Firm Scale and Scope: NielsenIQ and NielsenIQ–Compustat Samples



Notes: Same as Figure A3, comparing the full US NielsenIQ RMS sample with the subsample matched to Compustat. In the matched subsample, firms are defined at the level of their Compustat parent (Appendix A.1). Products are CMIE categories (see Section 2.1).

Figure A8: Firm Scope and Size: NielsenIQ–Compustat Sample



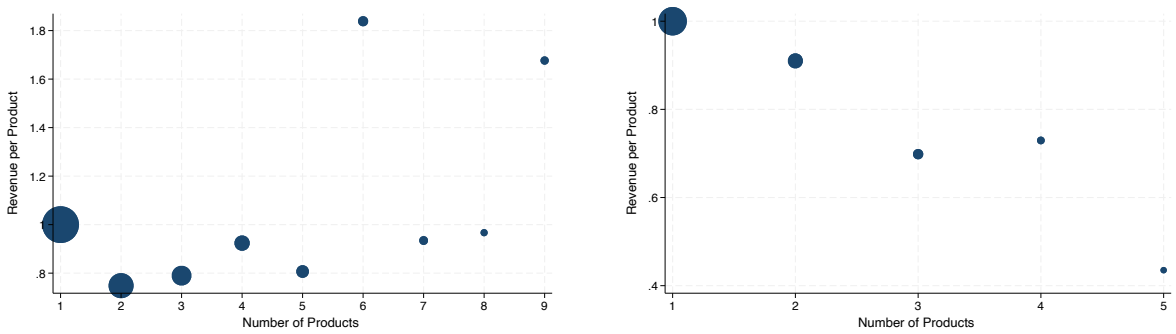
(a) Sales (b) Total assets

Notes: Binned scatter plots of the log number of products against log Compustat sales and total assets. Products are defined as UPCs.

A.8. Sample: India

Figure A9a plots the Indian data for Figure 2, where firms are defined at the business-group level in Prowess. I aggregate companies in each business group into a single firm and treat each company outside any group as its own firm. Figure A9b restricts the sample to CPG products. A firm's scope and revenue per product are computed over its CPG products only. Table A8 reports the regression in column (5) of Table A6 for the full Indian sample and for 2006–2019, the years covered by NielsenIQ. Figure A10 uses the ASI, with establishments in place of firms and products defined as ASICC codes rather than CMIE categories. These exercises address firm definition (business groups), sectoral coverage (CPG), sample period (Table A8), and firm selection (ASI) for India.

Figure A9: Firm Scale and Scope: India, Business Groups and CPG Products



(a) Business groups (b) CPG products

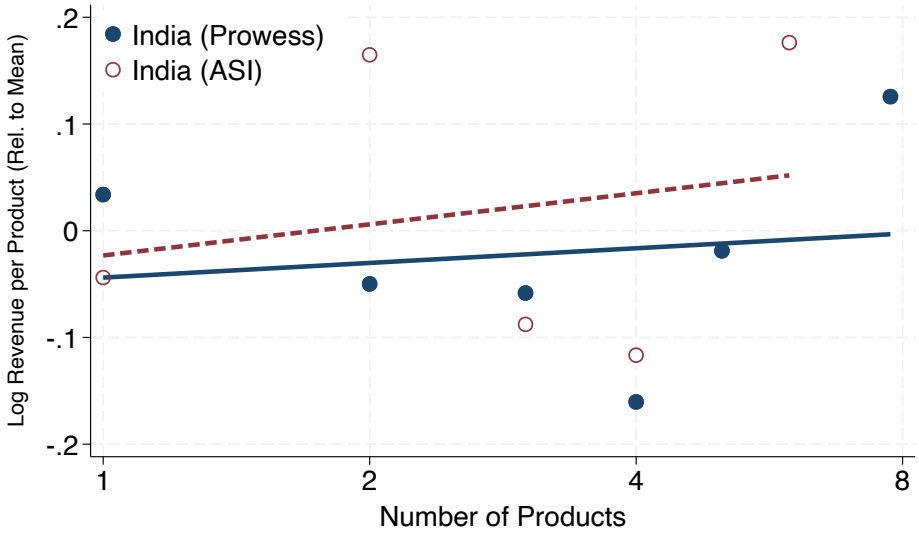
Notes: Same as Figure 2, for India only. Panel (a) defines firms as business groups. Panel (b) restricts the sample to CPG products. The number of products is top-coded at the 99th percentile of each sample (9 in panel (a), 5 in panel (b)). Products are CMIE categories (see Section 2.1).

Table A8: Revenue per Product and Scope: India, Full and NielsenIQ-Period Samples

	(1)	(2)
Log(Number of Products)	-0.001	0.051*
	(0.026)	(0.030)
Year FE	Yes	Yes
Industry FE	Yes	Yes
Sample	All	2006–2019
N	116573	62461
R^2	.0955	.0792

Notes: Column (1) uses the full Prowess sample; column (2) restricts it to the years covered by NielsenIQ RMS. Industry is 3-digit NIC. Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Figure A10: Establishment Scale and Scope: India, Annual Survey of Industries



Notes: Same as Figure A3, for India using the Annual Survey of Industries (ASI), with establishments in place of firms. The sample covers the 2001–2010 rounds, in which products are classified using the Annual Survey of Industries Commodity Classification (ASICC). For comparison, the figure also shows the full Prowess sample (1998–2019).

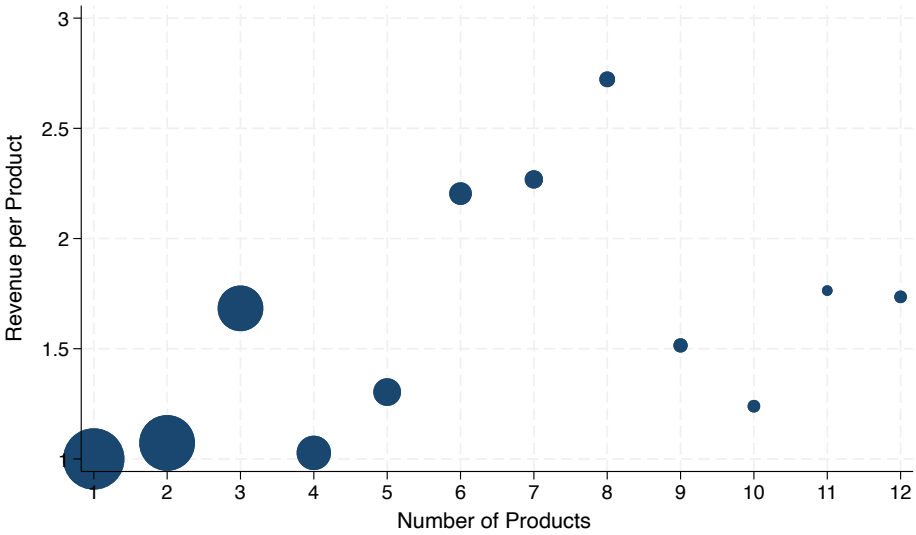
A.9. Comparison with Goldberg et al. (2010b)

Using Prowess data for 1989–2003, Goldberg et al. (2010b) find that Indian multi-product firms resemble their US counterparts in the cross section. In particular, they report a positive elasticity (0.43) of output per product with respect to the number of products. Figure A11 reproduces Figure 2 for India over 1998–2003, the years that overlap with their sample. Revenue per product is higher for the few firms with six to eight products, but the relationship remains far weaker than in the US.

A.10. Cost-Shock Instrument: First Stages and Reduced Forms

Table A9 reports the first stage, reduced form, IV, and OLS estimates behind columns (1) and (2) of Table 4. Table A10 reports the same for column (3), with the cost-shock exposure lagged one year, $B_{f,t-1}$, in place of $B_{f,t}$.

Figure A11: Firm Scale and Scope: India, Restricted Sample



Notes: Same as Figure 2, for India only, with the sample restricted to 1998–2003. The number of products is top-coded at the 99th percentile of this sample (12). Products are CMIE categories (see Section 2.1).

Table A9: Revenue per Product and Scope: Cost-Shock Instrument, Full Results

	Log # prod.	Avg. log revenue		
	(1) First stage	(2) Reduced form	(3) IV	(4) OLS
$B_{f,t}$	-11.832***	-5.628***		
	(0.629)	(1.230)		
Log # products			0.476***	0.802***
			(0.100)	(0.010)
Observations	139524	139524	139524	139524
K-P F			354.09	

Notes: Firm-year observations. Products are defined as UPCs. K-P F is the Kleibergen–Paap first-stage F statistic. Standard errors clustered by firm in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table A10: Revenue per Product and Scope: Lagged Cost-Shock Instrument

	Log # prod.	Avg. log revenue		
	(1) First stage	(2) Reduced form	(3) IV	(4) OLS
$B_{f,t-1}$	-15.116***	-3.029***		
	(0.637)	(1.066)		
Log # products			0.200***	0.772***
			(0.069)	(0.011)
Observations	132074	132074	132074	132074
K-P F			562.63	

Notes: Same as Table A9, with the cost-shock exposure lagged one year, $B_{f,t-1}$, in place of $B_{f,t}$.

Model Appendix

B.1. Firm-Level Derivations

B.1.1. Proof of Proposition 1

I state the proof for $d_f = 1$. The case $d_f = 0$ is the same with $\rho = 0$ wherever it appears, since the organizational constraint (11) under rival management is the constraint under non-rival management at $\rho = 0$, together with $P_m^0 = W$ and hence Ω_0 in place of Ω_1.

Substituting the demand (2) into problem (8), revenue from product i is $Ay_{i,f}^{\eta}$, so the firm solves

$$\Pi_f^1 = \max_{n_f, m_f, \{\ell_{i,f}, x_{i,f}, m_{i,f}\}_i} \int_0^{n_f} A\left(z_{i,f}\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta} di - W\int_0^{n_f}\ell_{i,f}\,di - P\int_0^{n_f} x_{i,f}\,di - P_m^1 m_f - \mathcal{C}_f(n_f),$$

subject to (11). Let λ be the multiplier on (11). The first-order conditions for the product-level inputs are

$$\begin{aligned}
[\ell_{i,f}]: &\quad W\ell_{i,f} = \eta(1-\alpha-\gamma)A\left(z_{i,f}\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta},\\
[x_{i,f}]: &\quad Px_{i,f} = \eta\gamma A\left(z_{i,f}\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta},\\
[m_{i,f}]: &\quad \lambda\, m_f^{\frac{\rho}{\rho-1}} m_{i,f}^{\frac{1}{1-\rho}} = \eta\alpha A\left(z_{i,f}\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta},
\end{aligned}$$

and the first-order condition for m_f gives $\lambda = P_m^1$. Since $\lambda > 0$, the organizational constraint holds with equality. Let

$$R_f = \int_0^{n_f} z_{i,f}^{\eta}\left(\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta} di.$$

Integrating the condition for $m_{i,f}$ across products and using $\int_0^{n_f} m_{i,f}^{\frac{1}{1-\rho}} di = m_f^{\frac{1}{1-\rho}}$ gives $\lambda m_f = \eta\alpha A R_f$. Integrating the conditions for $\ell_{i,f}$ and $x_{i,f}$, and substituting for λ,

$$W\ell_f = \eta(1-\alpha-\gamma)AR_f, \qquad Px_f = \eta\gamma AR_f, \qquad P_m^1 m_f = \eta\alpha AR_f,$$

so the managerial input is a share α of the firm's variable costs, irrespective of ρ, n_f, and d_f. Combining the conditions for $m_{i,f}$ and m_f,

$$m_{i,f} = \frac{\eta\alpha A}{P_m^1}R_f^{\rho}\left(z_{i,f}^{\eta}\left(\ell_{i,f}^{1-\alpha-\gamma}x_{i,f}^{\gamma}m_{i,f}^{\alpha}\right)^{\eta}\right)^{1-\rho}.$$

At $\rho = 0$ this takes the same form as the condition for manual labor; at $\rho = 1$ it gives $m_{i,f} = m_f$ for every product, so one input is used simultaneously across all of them.

Solve the three conditions jointly. The ratio of the conditions for $x_{i,f}$ and $\ell_{i,f}$ gives $x_{i,f} = \frac{W}{1-\alpha-\gamma}\frac{\gamma}{P}\ell_{i,f}$; eliminating $x_{i,f}$ from the condition for $\ell_{i,f}$ expresses $\ell_{i,f}$ as a power of $z_{i,f}^{\eta}m_{i,f}^{\alpha\eta}$;

substituting that into the condition for $m_{i,f}$ and collecting exponents yields $m_{i,f}$, and hence $\ell_{i,f}$ and $x_{i,f}$, in closed form. The resulting product-level allocations are

$$\ell_{i,f} = (A\eta/\Omega_1)^{\frac{1}{1-\eta}}\,\Omega_1 \frac{1-\alpha-\gamma}{W}\left(\int_0^{n_f} z_{j,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\,dj\right)^{\frac{\rho\alpha\eta}{1-\eta}} z_{i,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}},$$

$$x_{i,f} = (A\eta/\Omega_1)^{\frac{1}{1-\eta}}\,\Omega_1 \frac{\gamma}{P}\left(\int_0^{n_f} z_{j,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\,dj\right)^{\frac{\rho\alpha\eta}{1-\eta}} z_{i,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}},$$

$$m_{i,f} = (A\eta/\Omega_1)^{\frac{1}{1-\eta}}\,\Omega_1 \frac{\alpha}{P_m^1}\left(\int_0^{n_f} z_{j,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\,dj\right)^{\frac{\rho(1-\eta(1-\alpha))}{1-\eta}} z_{i,f}^{\frac{\eta(1-\rho)}{1-\eta(1-\rho\alpha)}},$$

where Ω_1 is the unit cost defined in Section 4.1. Integrating each across products, and using $\frac{\rho\alpha\eta}{1-\eta}+1=\frac{1-\eta(1-\rho\alpha)}{1-\eta}$,

$$\ell_f = (A\eta/\Omega_1)^{\frac{1}{1-\eta}}\,\Omega_1 \frac{1-\alpha-\gamma}{W}\left(\int_0^{n_f} z_{i,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\,di\right)^{\frac{1-\eta(1-\rho\alpha)}{1-\eta}},$$

and likewise for x_f and m_f, with $\frac{\gamma}{P}$ and $\frac{\alpha}{P_m^1}$ in place of $\frac{1-\alpha-\gamma}{W}$. Define firm-level productivity

$$Z_f = \left(\int_0^{n_f} z_{i,f}^{\frac{\eta}{1-\eta(1-\rho\alpha)}}\,di\right)^{\frac{1-\eta(1-\rho\alpha)}{\eta}},$$

which is the expression stated in the proposition for $d=1$, and for $d=0$ at $\rho=0$. The common factor in the firm-level allocations is then $Z_f^{\frac{\eta}{1-\eta}}$, so they are the factor demands reported in Section 4.1, and dividing the product-level allocations by them gives the within-firm shares in equation (15).

It remains to show that these allocations solve problem (13). Using the condition for $\ell_{i,f}$ to write $Ay_{i,f}^{\eta}$ in terms of $\ell_{i,f}$ and substituting the solution above, total revenue is

$$\int_0^{n_f} p_{i,f}y_{i,f}\,di = A\,(A\eta/\Omega_1)^{\frac{\eta}{1-\eta}}\,Z_f^{\frac{\eta}{1-\eta}} = A\left(Z_f\ell_f^{1-\alpha-\gamma}x_f^{\gamma}m_f^{\alpha}\right)^{\eta},$$

where the second equality follows from substituting the firm-level allocations. Total variable costs are $W\ell_f + Px_f + P_m^1 m_f = \eta A\,(A\eta/\Omega_1)^{\frac{\eta}{1-\eta}}\,Z_f^{\frac{\eta}{1-\eta}}$, so

$$\Pi_f^1 = (1-\eta)A\,(A\eta/\Omega_1)^{\frac{\eta}{1-\eta}}\,Z_f^{\frac{\eta}{1-\eta}} - \mathcal{C}_f(n_f),$$

and the first-order condition for scope is

$$f_{c,f}n_f^{\omega} = (1-\eta(1-\rho\alpha))\,(\eta/\Omega_1)^{\frac{\eta}{1-\eta}}\,A^{\frac{1}{1-\eta}}Z_f^{\frac{\eta}{1-\eta}}\left(\frac{z_{n,f}}{Z_f}\right)^{\frac{\eta}{1-\eta(1-\rho\alpha)}}.$$

Problem (13) delivers the same first-order conditions in ℓ_f, x_f, m_f, and n_f: its technology is Cobb–Douglas in firm-level inputs with unit cost Ω_1 and productivity Z_f, the only difference

being that n_f enters through Z_f. The two problems therefore have the same solution and the same value. ■

B.1.2. Scope Elasticity of Firm-Level Productivity

Section 4.2 states the elasticity of Z_f with respect to n_f under symmetry. For arbitrary $\{z_{i,f}\}_i$, differentiate Z_f as given in Proposition 1,

$$\frac{\partial \log Z_f}{\partial \log n_f} = \frac{1-\eta(1-d_f\rho\alpha)}{\eta} \cdot \frac{n_f z_{n,f}^{\frac{\eta}{1-\eta(1-d_f\rho\alpha)}}}{\int_0^{n_f} z_{j,f}^{\frac{\eta}{1-\eta(1-d_f\rho\alpha)}} dj} = \left(\frac{1-\eta}{\eta} + d_f\rho\alpha\right) \frac{s_{n,f}}{1/n_f},$$

where $s_{n,f}$ is the sales share of the marginal product, given by equation (15) evaluated at $i = n_f$. The elasticity under symmetry is scaled by the sales share of the marginal product relative to the uniform allocation $1/n_f$. If $\delta > 0$, then $s_{n,f}/(1/n_f) < 1$.

B.1.3. Productivity Gain from Adoption

Section 4.2 states $\Delta \log Z_f$ under symmetry. For arbitrary $\{z_{i,f}\}_i$, write Z_f as a power mean of $\{z_{i,f}\}_i$ and expand it to second order around uniform sales shares, $s_{i,f} = 1/n_f$. With $\text{HHI}_f = \int_0^{n_f} s_{i,f}^2 di$ the within-firm Herfindahl–Hirschman index over the sales shares in equation (15),

$$\Delta \log Z_f \approx \rho\alpha \log n_f - \frac{\rho\alpha}{2}\left(\frac{1-\eta(1-\rho\alpha)}{1-\eta}\right)(n_f \text{HHI}_f - 1),$$

where $n_f\text{HHI}_f - 1 = n_f \int_0^{n_f} (s_{i,f} - 1/n_f)^2 \, di$ is the dispersion of sales shares around the uniform allocation. It is zero at $\delta = 0$, which gives the expression in the text, and the gain from adoption falls as sales become more concentrated within the firm.

B.1.4. Proof of Proposition 2

I state the proof for $d_f = 1$; the case $d_f = 0$ is the same with $\rho = 0$ and $P_m^0 = W$. It is useful to study the dual problem. Conditional on n_f, d_f, and output levels $\{y_{j,f}\}_j$, the firm solves

$$\min_{\{\ell_{i,f}, x_{i,f}, m_{i,f}\}_i, m_f} W \int_0^{n_f} \ell_{i,f}\, di + P \int_0^{n_f} x_{i,f}\, di + P_m^1 m_f \quad \text{subject to} \quad (10) \text{ and } (11).$$

Let $MC_{i,f}$ be the multiplier on the technology (10) for product i, that is, the marginal cost of producing $y_{i,f}$, and let λ be the multiplier on the organizational constraint (11). The first-order conditions are

$$W\ell_{i,f} = MC_{i,f}(1-\alpha-\gamma)y_{i,f}, \qquad Px_{i,f} = MC_{i,f}\gamma y_{i,f}, \qquad \lambda\, m_f^{\frac{\rho}{\rho-1}} m_{i,f}^{\frac{1}{1-\rho}} = MC_{i,f}\alpha y_{i,f},$$

with $\lambda = P_m^1$ from the condition for m_f. Integrating the third condition across products and using the organizational constraint gives $P_m^1 m_f = \alpha \int_0^{n_f} MC_{j,f} y_{j,f}\, dj$. Substituting this back into the same condition,

$$\frac{m_{i,f}}{m_f} = \left(\frac{MC_{i,f} y_{i,f}}{\int_0^{n_f} MC_{j,f} y_{j,f}\, dj} \right)^{1-\rho}, \qquad \text{so} \qquad m_{i,f} = \frac{\alpha}{P_m^1} MC_{i,f} y_{i,f} \left(\frac{MC_{i,f} y_{i,f}}{\int_0^{n_f} MC_{j,f} y_{j,f}\, dj} \right)^{-\rho}.$$

Substituting this solution for $m_{i,f}$, together with the conditions for $\ell_{i,f}$ and $x_{i,f}$, into the technology (10) and collecting terms gives

$$MC_{i,f} = \frac{\Omega_1}{z_{i,f}} \left(\frac{MC_{i,f} y_{i,f}}{\int_0^{n_f} MC_{j,f} y_{j,f}\, dj} \right)^{\rho\alpha}, \tag{B.1}$$

where Ω_1 is the unit cost defined in Section 4.1. Marginal cost depends on the cost share of product i: when products are symmetric that share is $1/n_f$ and $MC_{i,f} = (\Omega_1 / z_{i,f})\, n_f^{-\rho\alpha}$.

Equation (B.1) is implicit in $MC_{i,f}$. To solve it, multiply through by $y_{i,f}$ and rearrange,

$$MC_{i,f} y_{i,f} = \left(\frac{y_{i,f}}{z_{i,f}} \right)^{\frac{1}{1-\rho\alpha}} \Omega_1^{\frac{1}{1-\rho\alpha}} \left(\int_0^{n_f} MC_{j,f} y_{j,f}\, dj \right)^{-\frac{\rho\alpha}{1-\rho\alpha}}.$$

Integrating both sides across products and solving for the integral gives total cost,

$$\int_0^{n_f} MC_{j,f} y_{j,f}\, dj = \Omega_1 \left(\int_0^{n_f} \left(\frac{y_{j,f}}{z_{j,f}} \right)^{\frac{1}{1-\rho\alpha}} dj \right)^{1-\rho\alpha},$$

and substituting it back delivers the expression in the proposition. Setting $\rho = 0$ gives $MC_{i,f} = \Omega_0 / z_{i,f}$, the case $d_f = 0$. ■

Two implications are used in the text. First, combining the condition for $\ell_{i,f}$ above with its counterpart in the firm's problem (8) gives $MC_{i,f} y_{i,f} = \eta A y_{i,f}^{\eta}$, so $MC_{i,f} = \eta p_{i,f}$: the markup is the constant $1/\eta$, and economies of scope appear entirely in marginal cost rather than in the markup. Second, cost shares and sales shares therefore coincide product by product, so substituting the optimal output into the proposition gives $MC_{i,f} = (\Omega_1 / z_{i,f})\, (z_{i,f}/Z_f)^{\frac{\rho\alpha\eta}{1-\eta(1-\rho\alpha)}}$, as stated in Section 4.2.

B.1.5. Marginal Cost Decomposition

Section 4.2 decomposes $\Delta \log MC_{i,f}$ under symmetry. For arbitrary $\{z_{i,f}\}_i$, Proposition 2 gives, holding n_f and $\{y_{j,f}\}_j$ fixed,

$$\Delta \log MC_{i,f} = \underbrace{\rho\alpha \log s_{i,f}}_{\text{cost savings}} - \underbrace{(\log \Omega_0 - \log \Omega_1)}_{\text{cost advantage}},$$

where $s_{i,f}$ is the cost share of product i, which equals its sales share. Cost savings are larger for products with smaller shares, and under symmetry $s_{i,f} = 1/n_f$ returns the expression in

the text.

At the firm level, write $q_{i,f} = y_{i,f}/z_{i,f}$ for effective output. Total cost is $C_f = \Omega_1 \left(\int_0^{n_f} q_{i,f}^{\frac{1}{1-\rho\alpha}} di \right)^{1-\rho\alpha}$, and the cost of the same output bundle with rival management is $C_f^{\text{riv}} = \Omega_0 \int_0^{n_f} q_{i,f} di$. Their ratio separates the two terms,

$$C_f = C_f^{\text{riv}} \frac{\Omega_1}{\Omega_0} \mathbf{\Phi}_f, \qquad \mathbf{\Phi}_f = \left(\int_0^{n_f} s_{i,f}^{1-\rho\alpha} di \right)^{-1},$$

so $-\log \mathbf{\Phi}_f$ is the firm-level cost saving. Expanding $s_{i,f}$ to second order around the uniform allocation,

$$-\log \mathbf{\Phi}_f \approx \rho\alpha \log n_f - \frac{\rho\alpha(1-\rho\alpha)}{2} \left(n_f \text{HHI}_f - 1 \right).$$

By the Cauchy–Schwarz inequality, $n_f \text{HHI}_f \geq 1$, with equality if and only if products are symmetric, in which case $\mathbf{\Phi}_f = n_f^{-\rho\alpha}$ exactly. Cost savings therefore come from two margins: the number of products, over which the managerial input is amortized, and the breadth of sales across them, since concentration limits how widely the input is shared.

The cost saving and the productivity gain in Appendix B.1.3 agree in the scope term and differ only in the coefficient on concentration,

$$\Delta \log Z_f + \log \mathbf{\Phi}_f \approx -\frac{(\rho\alpha)^2}{2(1-\eta)} \left(n_f \text{HHI}_f - 1 \right).$$

They are not the same object: $\mathbf{\Phi}_f$ compares costs at a fixed output bundle, while $\Delta \log Z_f$ allows the firm to re-optimize its output mix.

B.1.6. Scope and Adoption in the Simplified Model

Throughout this subsection $\delta = 0$, $\sigma_c = 0$ (so $f_{c,f} = f_c$), and $f_e = 0$, as in Section 4.3. Under $\delta = 0$, $z_{i,f} = z_f$ and $Z_f = z_f n_f^{\frac{1-\eta(1-d_f\rho\alpha)}{\eta}}$.

Scope. Substituting Z_f into the first-order condition for scope derived in Appendix B.1 and collecting the powers of n_f gives equation (19), that is,

$$n_f^0 = \left(\frac{(1-\eta)\,(\eta/\Omega_0)^{\frac{\eta}{1-\eta}}\, A^{\frac{1}{1-\eta}} z_f^{\frac{\eta}{1-\eta}}}{f_c} \right)^{\frac{1}{\omega}}, \qquad n_f^1 = \left(\frac{(1-\eta(1-\rho\alpha))\,(\eta/\Omega_1)^{\frac{\eta}{1-\eta}}\, A^{\frac{1}{1-\eta}} z_f^{\frac{\eta}{1-\eta}}}{f_c} \right)^{\frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta}}}.$$

The exponent in n_f^1 requires $\omega > \frac{\rho\alpha\eta}{1-\eta}$. Without it, the marginal benefit of scope rises faster than its marginal cost and n_f^1 is not interior. Differentiating in $\log z_f$ gives equation (20).

Profits. Substituting the scope policies into $\Pi_f^d = (1-\eta)A\left(A\eta/\Omega_d\right)^{\frac{\eta}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} - \mathcal{C}_f(n_f)$ from Appendix B.1,

$$\Pi_f^0 = \frac{\omega f_c}{1+\omega}\left(n_f^0\right)^{1+\omega}, \qquad \Pi_f^1 = \frac{\left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right) f_c}{(1+\omega)\left(1+\frac{\rho\alpha\eta}{1-\eta}\right)}\left(n_f^1\right)^{1+\omega}.$$

The restriction $\omega > \frac{\rho\alpha\eta}{1-\eta}$ is also what makes $\Pi_f^1 > 0$. Their elasticities with respect to core productivity are

$$\frac{\partial \log \Pi_f^0}{\partial \log z_f} = \frac{\eta}{1-\eta}\frac{1+\omega}{\omega}, \qquad \frac{\partial \log \Pi_f^1}{\partial \log z_f} = \frac{\eta}{1-\eta}\frac{1+\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}},$$

so profits grow faster with core productivity under adoption.

Proof of Proposition 3

Write Π_f^1 in terms of the scope the firm would choose without adopting. Taking the ratio of the two policies above and substituting,

$$\Pi_f^1 = \frac{\left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right) f_c}{(1+\omega)\left(1+\frac{\rho\alpha\eta}{1-\eta}\right)}\left(\frac{1-\eta(1-\rho\alpha)}{1-\eta}\left(\frac{\Omega_0}{\Omega_1}\right)^{\frac{\eta}{1-\eta}}\right)^{\frac{1+\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}}\left(n_f^0\right)^{\frac{\omega(1+\omega)}{\omega - \frac{\rho\alpha\eta}{1-\eta}}},$$

while Π_f^0 is proportional to $\left(n_f^0\right)^{1+\omega}$. Under $\omega > \frac{\rho\alpha\eta}{1-\eta}$, the exponent on n_f^0 is larger for Π_f^1, so with $f_e = 0$ the two cross once and $\Pi_f^1 \geq \Pi_f^0$ if and only if $n_f^0 \geq \bar{n}$, where

$$\bar{n} = \left(\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}\right)^{\frac{\omega - \frac{\rho\alpha\eta}{1-\eta}}{\frac{\rho\alpha\eta}{1-\eta}(1+\omega)}}\left(1+\frac{\rho\alpha\eta}{1-\eta}\right)^{-\frac{1+\frac{\rho\alpha\eta}{1-\eta}}{\frac{\rho\alpha\eta}{1-\eta}(1+\omega)}}\left(\frac{\Omega_1}{\Omega_0}\right)^{\frac{\eta}{1-\eta}\frac{1}{\frac{\rho\alpha\eta}{1-\eta}}}.$$

Since n_f^0 is increasing in z_f, the rule is equivalent to $z_f \geq \bar{z}$, with

$$\log \bar{z} = \text{Const}(\rho) + \frac{1-\eta}{\eta}\log\left(\frac{f_c}{A^{\frac{1}{1-\eta}}}\right) + \log\left(\frac{\Omega_0}{\eta}\right) + \frac{\omega(1-\eta)}{\eta\rho\alpha}\log\left(\frac{\Omega_1}{\Omega_0}\right),$$

where $\text{Const}(\rho)$ collects the terms that depend on ρ, ω, and η alone. The coefficient on $\log(\Omega_1/\Omega_0)$ is positive, so $\bar{z}$ decreases with the cost advantage $\log(\Omega_0/\Omega_1)$. ■

Jumps at the Threshold. Taking the log ratio of the two scope policies,

$$\log\frac{n_f^1}{n_f^0} = \frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta}}\left(\frac{\rho\alpha\eta}{1-\eta}\log n_f^0 + \log\left(1+\frac{\rho\alpha\eta}{1-\eta}\right) - \frac{\eta}{1-\eta}\log\frac{\Omega_1}{\Omega_0}\right).$$

Evaluating at $n_f^0 = \bar{n}$ and substituting the expression for $\bar{n}$ above, the terms in Ω_1/Ω_0 cancel and

$$\log \frac{n_f^1(\bar{z})}{\bar{n}} = \frac{1}{1+\omega} \log \left(\frac{\omega \left(1 + \frac{\rho\alpha\eta}{1-\eta}\right)}{\omega - \frac{\rho\alpha\eta}{1-\eta}} \right),$$

which depends only on ω and $\frac{\rho\alpha\eta}{1-\eta}$ and is increasing in ρ. The jump in firm-level productivity follows from $Z_f = z_f n_f^{\frac{1-\eta(1-d_f\rho\alpha)}{\eta}}$ evaluated at $z_f = \bar{z}$ under each policy. Away from the threshold, differencing $\log Z_f^1$ and $\log Z_f^0$ and using equation (20) gives equation (21).

B.2. Scope, Adoption, and Firm-Size Distribution

B.2.1. Exact Distribution in the Simplified Model

Maintain the simplifying assumptions of Section 4.3: $\delta = 0$, $\sigma_c = 0$, and $f_e = 0$. Within each regime, scope is given by equation (19). Let $\bar{n} \equiv n_f^0(\bar{z})$ and $\bar{n}^1 \equiv n_f^1(\bar{z})$ be scope just below and at the adoption threshold, and let $\bar{d} \equiv (z_{\min}/\bar{z})^{\zeta}$ be the adoption share.

Proposition 7. *In the simplified model, suppose $\bar{z} > z_{\min}$. Then*

$$\mathbb{P}(n_f > x) = \begin{cases} \bar{d}\,(x/\bar{n})^{-\zeta_n^{(0)}} & \text{if } x < \bar{n}, \\ \bar{d} & \text{if } \bar{n} \le x \le \bar{n}^1, \\ \bar{d}\,(x/\bar{n}^1)^{-\zeta_n^{(1)}} & \text{if } x > \bar{n}^1. \end{cases} \tag{B.2}$$

Moreover: (i) $\zeta_n^{(1)} < \zeta_n^{(0)}$, and $\zeta_n^{(1)}$ decreases with ρ; (ii) no firm has scope in $(\bar{n}, \bar{n}^1)$, and the length of this gap, $\log(\bar{n}^1/\bar{n})$, increases with ρ; (iii) $\bar{d}$ increases with ρ and with $\log(\Omega_0/\Omega_1)$.

Proof. For $x < \bar{n}$, every adopter has scope above $\bar{n}^1 > x$, so $\mathbb{P}(n_f > x) = \mathbb{P}(n_f^0 > x)$, with n_f^0 evaluated at every firm. For $\bar{n} \le x \le \bar{n}^1$, $n_f > x$ if and only if $z_f \ge \bar{z}$. For $x > \bar{n}^1$, every firm with $n_f > x$ adopts, so $\mathbb{P}(n_f > x) = \mathbb{P}(n_f^1 > x)$. In each segment, n_f^d is a power of z_f with exponent given by equation (20), which gives the tail indices in equation (22). Part (i) follows from equation (22). Part (ii) follows from the jump in scope at the threshold (Section 4.3),

$$\log \frac{\bar{n}^1}{\bar{n}} = \frac{1}{1+\omega} \log \left(\frac{\omega \left(1 + \frac{\rho\alpha\eta}{1-\eta}\right)}{\omega - \frac{\rho\alpha\eta}{1-\eta}} \right),$$

which increases with ρ. Part (iii) follows from Proposition 3. □

Sales. Within each regime, S_f^d is a power of z_f, and $\Pi_f^d = (1-\eta)\frac{\omega - d\frac{\rho\alpha\eta}{1-\eta}}{1+\omega} S_f^d$. Let $\bar{S}^0 \equiv S_f^0(\bar{z})$ and $\bar{S}^1 \equiv S_f^1(\bar{z})$. The distribution of sales has the same three segments as equation (B.2), with

tail indices $\zeta_n^{(d)}/(1+\omega)$ and the gap $(\bar{S}^0, \bar{S}^1)$. Equating Π_f^0 and Π_f^1 at $\bar{z}$ gives the jump in sales,

$$\log \frac{\bar{S}^1}{\bar{S}^0} = \log \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}},$$

which, like the jump in scope, does not depend on Ω_0/Ω_1 and increases with ρ.

Adoption Costs. The two costs of adoption act on the distribution differently. A fall in Ω_1, holding Ω_0 fixed, lowers $\bar{z}$ and raises adopters' scope at every z_f: the adopter segment begins farther left and shifts up. With $f_e > 0$, the threshold solves $\Pi_f^1 - f_e = \Pi_f^0$. A fall in f_e lowers $\bar{z}$ but leaves adopters' policies unchanged: the adopter segment begins farther left, but the far tail is unchanged. The jump formulas above assume $f_e = 0$.

B.2.2. Proof of Proposition 4

Step 1: Scope. With core competency, adopters' firm-level productivity is $Z_f = z_f \, \mathcal{A}(n_f)^{\frac{1-\eta(1-\rho\alpha)}{\eta}}$, where $\mathcal{A}(n) = \int_0^n (1+i)^{-\frac{\delta\eta}{1-\eta(1-\rho\alpha)}} \, di$. The first-order condition for scope is

$$\Psi(n_f) = (1-\eta(1-\rho\alpha)) \left(\frac{\eta}{\Omega_1}\right)^{\frac{\eta}{1-\eta}} A^{\frac{1}{1-\eta}} \frac{z_f^{\frac{\eta}{1-\eta}}}{f_{c,f}}, \qquad \Psi(n) = n^{\omega} \mathcal{A}(n)^{-\frac{\rho\alpha\eta}{1-\eta}} (1+n)^{\frac{\delta\eta}{1-\eta(1-\rho\alpha)}}.$$

Since $\frac{d\log \mathcal{A}(n)}{d\log n} \in (0,1]$, $\frac{d\log \Psi(n)}{d\log n} \geq \omega - \frac{\rho\alpha\eta}{1-\eta} > 0$, so Ψ is strictly increasing and scope depends on $(z_f, f_{c,f})$ only through $z_f^{\frac{\eta}{1-\eta}}/f_{c,f}$. Since $\frac{\delta\eta}{1-\eta(1-\rho\alpha)} < \frac{\delta\eta}{1-\eta} < 1$, $\mathcal{A}(n)$ grows as $n^{1-\frac{\delta\eta}{1-\eta(1-\rho\alpha)}}$, and $\Psi(n)$ grows as $n^{\omega - \frac{(\rho\alpha-\delta)\eta}{1-\eta}}$. Hence, as $z_f \to \infty$,

$$n_f = C \left(\frac{z_f^{\frac{\eta}{1-\eta}}}{f_{c,f}}\right)^{\frac{1}{\omega - \frac{(\rho\alpha-\delta)\eta}{1-\eta}}} (1+o(1))$$

for a constant $C > 0$.

Step 2: Tail of Scope. By Breiman's lemma, if $f_{c,f}$ is independent of z_f and has all moments, as the log-normal does,

$$\mathbb{P}\left(\frac{z_f^{\frac{\eta}{1-\eta}}}{f_{c,f}} > r\right) = \mathbb{E}\left[f_{c,f}^{-\zeta\frac{1-\eta}{\eta}}\right] z_{\min}^{\zeta} \, r^{-\zeta\frac{1-\eta}{\eta}} \, (1+o(1)), \qquad r \to \infty.$$

Substituting $r = (x/C)^{\omega - \frac{(\rho\alpha-\delta)\eta}{1-\eta}}$ from Step 1 gives $\mathbb{P}(n_f > x) \propto x^{-\zeta_n^{(1)}} (1+o(1))$, with $\zeta_n^{(1)}$ as in equation (23).

Step 3: Adopters Occupy the Tail. Adopter profits grow with z_f at a strictly higher power than non-adopter profits, so for every $f_{c,f}$ the gain from adoption exceeds f_e for z_f large enough, and a positive measure of firms adopts. A non-adopter has $z_f^{\frac{\eta}{1-\eta}}/f_{c,f}$ below an adoption threshold that is bounded by a power of $f_{c,f}$ (Section 4.3), so its scope is bounded by a power of $f_{c,f}$. The tail of a power of a log-normal variable vanishes faster than any power of x. Non-adopters therefore do not affect the tail index, and $\mathbb{P}(d_f = 1 \mid n_f > x) \to 1$.

Step 4: Sales and Employment. Sales are $S_f = A^{\frac{1}{1-\eta}} \left(\frac{\eta}{\Omega_1}\right)^{\frac{\eta}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}}$, which grows as $z_f^{\frac{\eta}{1-\eta}} n_f^{1+\frac{(\rho\alpha-\delta)\eta}{1-\eta}}$. Substituting Step 1, S_f grows as a power $\frac{\eta}{1-\eta} \cdot \frac{1+\omega}{\omega - \frac{(\rho\alpha-\delta)\eta}{1-\eta}}$ of z_f, times a power of $f_{c,f}$. Sales are not a function of $z_f^{\frac{\eta}{1-\eta}}/f_{c,f}$ alone, so Step 2 does not apply directly. Bounding $\mathbb{P}(S_f > x)$ above and below by the tails of $(1 \pm \varepsilon)$ times this power, plus terms that are tails of powers of $f_{c,f}$, and applying Breiman's lemma gives the tail index $\zeta_n^{(1)}/(1+\omega)$. Employment is proportional to sales within each regime.

Step 5: No Adoption. With $\rho = 0$ and Ω_0 in place of Ω_1, $\Psi(n) = n^\omega (1+n)^{\frac{\delta\eta}{1-\eta}}$, which grows as $n^{\omega + \frac{\delta\eta}{1-\eta}}$. Steps 1, 2, and 4 then give $\zeta_n^{(0)}$ and $\zeta_n^{(0)}/(1+\omega)$; the sales result uses $\frac{\delta\eta}{1-\eta} < 1$. Subtracting gives $\zeta_n^{(0)} - \zeta_n^{(1)} = \zeta\rho\alpha$. □

B.2.3. Strong Core Competency

If $\frac{\delta\eta}{1-\eta(1-\rho\alpha)} > 1$, $\mathcal{A}(n)$ is bounded: products far from the core contribute little to firm-level productivity. The same steps give

$$\zeta_n^{(1)} = \zeta \frac{1-\eta}{\eta} \left(\omega + \frac{\delta\eta}{1-\eta(1-\rho\alpha)}\right), \qquad \zeta_S^{(1)} = \zeta \frac{1-\eta}{\eta}.$$

Scope no longer affects sales in the tail, so ρ does not affect the tail of sales, and affects the tail of scope only through $\frac{\delta\eta}{1-\eta(1-\rho\alpha)}$. Adopters and non-adopters then have sales that scale with z_f at the same power, and adopters occupy the tail only if non-rival management raises the level of sales; for large δ this fails and no firm adopts. The calibrated model satisfies $\frac{\delta\eta}{1-\eta} < 1$ (Section 5.3).

B.3. Identification Details

B.3.1. Bias in the Naive Regression

Set $\delta = 0$ and omit the time subscript. Start from the definition of the ordinary least squares coefficient in specification (25), ignoring the fixed effect, and substitute equation (24),

$$\begin{aligned}\beta &= \frac{\mathbb{C}\text{ov}\left(\log \frac{S_f}{n_f}, \log n_f\right)}{\mathbb{V}\text{ar}\left(\log n_f\right)} \\ &= \frac{\eta}{1-\eta}\log\left(\frac{\Omega_0}{\Omega_1}\right)\frac{\mathbb{C}\text{ov}\left(d_f, \log n_f\right)}{\mathbb{V}\text{ar}\left(\log n_f\right)} + \frac{\rho\alpha\eta}{1-\eta}\frac{\mathbb{C}\text{ov}\left(d_f \log n_f, \log n_f\right)}{\mathbb{V}\text{ar}\left(\log n_f\right)} + \frac{\eta}{1-\eta}\frac{\mathbb{C}\text{ov}\left(\log z_f, \log n_f\right)}{\mathbb{V}\text{ar}\left(\log n_f\right)}.\end{aligned}$$

The three terms are the regime-specific intercept, which carries Ω_1 for adopters and Ω_0 otherwise, the regime-specific slope, and unobserved core productivity.

Case 1: $d_f = 1$ for all f. The first term is zero and the second is $\frac{\rho\alpha\eta}{1-\eta}$. Using equation (19) for the third,

$$\begin{aligned}\beta &= \frac{\rho\alpha\eta}{1-\eta} + \left(\frac{\eta}{1-\eta}\right)^2 \frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta}} \frac{\mathbb{V}\text{ar}\left(\log z_f\right)}{\mathbb{V}\text{ar}\left(\log n_f\right)} \\ &= \frac{\rho\alpha\eta}{1-\eta} + \left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right) \frac{\left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f\right)}{\left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f\right) + \mathbb{V}\text{ar}\left(\log f_{c,f}\right)} \\ &= \frac{\rho\alpha\eta}{1-\eta} + \left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right) R_z^2 = \omega R_z^2 + \frac{\rho\alpha\eta}{1-\eta}\left(1 - R_z^2\right),\end{aligned}$$

where R_z^2 is the share of the variance of $\log n_f$ explained by $\log z_f$. At $R_z^2 = 1$, the slope is ω and nothing containing scope economies is identified, which is why variation in $f_{c,f}$ is needed.

Case 2: $d_f = 0$ for all f. The first two terms are zero and

$$\beta = \omega \frac{\left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f\right)}{\left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f\right) + \mathbb{V}\text{ar}\left(\log f_{c,f}\right)} = \omega R_z^2,$$

still positive, and smaller than in Case 1 at the same R_z^2.

Case 3: $\bar{d} \in (0,1)$ and $\bar{d}(1-\bar{d}) \approx 0$. Write

$$\Sigma_d \equiv \left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f \mid d_f = d\right) + \mathbb{V}\text{ar}\left(\log f_{c,f} \mid d_f = d\right), \qquad R_{z,d}^2 = \frac{\left(\frac{\eta}{1-\eta}\right)^2 \mathbb{V}\text{ar}\left(\log z_f \mid d_f = d\right)}{\Sigma_d},$$

so that $\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 1\right) = \left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right)^{-2}\Sigma_1$ and $\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 0\right) = \omega^{-2}\Sigma_0$, and let

$$\hat{d} \equiv \frac{\bar{d}\,\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 1\right)}{\bar{d}\,\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 1\right) + \left(1-\bar{d}\right)\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 0\right)}$$

denote the share of $\mathbb{V}\mathrm{ar}\left(\log n_f\right)$ contributed by adopters.

The first term of β is purely between-regime, since $\mathbb{C}\mathrm{ov}\left(d_f, \log n_f\right) = \bar{d}(1-\bar{d})\left(\mathbb{E}\left(\log n_f \mid d_f = 1\right) - \mathbb{E}\left(\log n_f \mid d\right.\right.$ so it is deleted by $\bar{d}(1-\bar{d}) \approx 0$. For the second term, decomposing the covariance and the variance into within- and between-regime components and deleting the latter,

$$\frac{\mathbb{C}\mathrm{ov}\left(d_f \log n_f, \log n_f\right)}{\mathbb{V}\mathrm{ar}\left(\log n_f\right)} \approx \hat{d}.$$

This does not deliver $\bar{d}$: that would additionally require $\mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 1\right) = \mathbb{V}\mathrm{ar}\left(\log n_f \mid d_f = 0\right)$, which fails because $\frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta}} > \frac{1}{\omega}$. Treating the third term the same way,

$$\frac{\eta}{1-\eta}\frac{\mathbb{C}\mathrm{ov}\left(\log z_f, \log n_f\right)}{\mathbb{V}\mathrm{ar}\left(\log n_f\right)} \approx \left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right)\hat{d}R_{z,1}^2 + \omega\left(1-\hat{d}\right)R_{z,0}^2.$$

Collecting the two,

$$\beta \approx \underbrace{\hat{d}\left[\frac{\rho\alpha\eta}{1-\eta} + \left(\omega - \frac{\rho\alpha\eta}{1-\eta}\right)R_{z,1}^2\right]}_{\text{Case 1}} + \left(1-\hat{d}\right)\underbrace{\left[\omega R_{z,0}^2\right]}_{\text{Case 2}},$$

a convex combination of the all-adopt and none-adopt slopes, each evaluated at its own $R_{z,d}^2$, with weights given by the variance shares rather than the population shares. In either regime ω is a nuisance that biases β upward.

Admissibility of $\hat{d} \approx \bar{d}$. When $\Sigma_1 = \Sigma_0$,

$$\hat{d} = \frac{\bar{d}}{\bar{d} + \left(1-\bar{d}\right)\left(\frac{\omega - \frac{\rho\alpha\eta}{1-\eta}}{\omega}\right)^2}, \qquad \hat{d} - \bar{d} = 2\bar{d}\left(1-\bar{d}\right)\frac{\frac{\rho\alpha\eta}{1-\eta}}{\omega} + O\left(\left(\frac{\frac{\rho\alpha\eta}{1-\eta}}{\omega}\right)^2\right),$$

so $\hat{d} > \bar{d}$: adopters have the more dispersed scope and are overweighted. The gap is second order in $\frac{\rho\alpha\eta}{(1-\eta)\omega}$ and vanishes under $\bar{d}(1-\bar{d}) \approx 0$, so imposing $\hat{d} = \bar{d}$, as equation (28) does, is a legitimate further approximation. It is separate from the deletion of the between-regime terms and is unavailable when $\Sigma_1 \neq \Sigma_0$. Imposing it also gives $R_{z,1}^2 = R_{z,0}^2 \equiv R_z^2$, so that equation (28) collapses to

$$\beta \approx \omega R_z^2 + \frac{\rho\alpha\eta}{1-\eta}\bar{d}\left(1 - R_z^2\right),$$

an interpolation between Cases 1 and 2 by the adoption rate. Only $1 - R_z^2$, the share of scope dispersion not generated by core productivity, carries the scope economies.

Case 4: $\bar{d} \in (0,1)$ Without the Approximation. Keep $\delta = 0$ and drop $\bar{d}(1-\bar{d}) \approx 0$. Write $\Delta w \equiv \mathbb{E}(w_f \mid d_f = 1) - \mathbb{E}(w_f \mid d_f = 0)$ and let

$$\varsigma_f \equiv \frac{\eta}{1-\eta} \log z_f - \log f_{c,f}$$

be the composite profit shifter, so that the scope policy in equation (19) reads $\log n_f = \frac{1}{\omega - d_f \frac{\rho\alpha\eta}{1-\eta}} \left(\text{Const}_{d_f} + \varsigma_f\right)$ and Proposition 3 reads $d_f = \mathbf{1}\{\varsigma_f \geq \omega \log \bar{n} - \text{Const}_0\}$: adoption is a threshold in the same index that generates scope. Scope therefore rises in ς_f with an upward jump at the cutoff, so $\Delta \log n_f > 0$ by construction and the approximation used in Case 3 is unavailable.

Under selection, $\log z_f$ and $\log f_{c,f}$ are correlated within regime, so define

$$R^2_{z,d} \equiv \frac{\frac{\eta}{1-\eta}\mathbb{C}\text{ov}(\log z_f, \varsigma_f \mid d_f = d)}{\mathbb{V}\text{ar}(\varsigma_f \mid d_f = d)}, \qquad R^2_{f,d} \equiv \frac{-\mathbb{C}\text{ov}(\log f_{c,f}, \varsigma_f \mid d_f = d)}{\mathbb{V}\text{ar}(\varsigma_f \mid d_f = d)},$$

the coefficients from projecting each component of ς_f on ς_f. Since the two components sum to ς_f identically, $R^2_{z,d} + R^2_{f,d} = 1$, which establishes $R^2_{z,d} \in [0,1]$ under arbitrary selection. When $\mathbb{C}\text{ov}(\log z_f, \log f_{c,f} \mid d_f = d) = 0$, this reduces to the expression in Case 3.

Let β_1 and β_0 be the Case 1 and Case 2 slopes, which remain the within-regime slopes, and let

$$\beta_b \equiv \frac{\Delta \log \frac{S_f}{n_f}}{\Delta \log n_f}, \qquad \vartheta \equiv \frac{\bar{d}\,\mathbb{V}\text{ar}(\log n_f \mid d_f = 1) + \left(1-\bar{d}\right)\mathbb{V}\text{ar}(\log n_f \mid d_f = 0)}{\mathbb{V}\text{ar}(\log n_f)},$$

so that $1 - \vartheta = \bar{d}(1-\bar{d})\left(\Delta \log n_f\right)^2 / \mathbb{V}\text{ar}(\log n_f)$. Decomposing the covariance in the definition of β into within- and between-regime parts and dividing by $\mathbb{V}\text{ar}(\log n_f)$,

$$\beta = \vartheta \left[\hat{d}\beta_1 + \left(1 - \hat{d}\right)\beta_0\right] + (1-\vartheta)\,\beta_b. \tag{B.3}$$

This is exact. Case 3 is the special case $\vartheta = 1$, and the intercept term deleted there returns inside β_b. The two weights split $\mathbb{V}\text{ar}(\log n_f)$ at two levels: ϑ is the within-regime block as a share of the total, and $\hat{d}$ is the adopters' share of that block. Equal scope dispersion across regimes would be needed for $\hat{d} = \bar{d}$ and equal regime means of $\log n_f$ for $\vartheta = 1$; the first is ruled out by $\frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta}} > \frac{1}{\omega}$ and the second by the threshold in ς_f.

The between-regime slope β_b is the slope through the adopter and non-adopter means, which pooled least squares uses because it does not condition on d_f. At $\delta = 0$ the marginal product's revenue is S_f/n_f, so the scope first-order condition reads $\log \frac{S_f}{n_f} = \widetilde{\text{Const}}_{d_f} + \omega \log n_f + \log f_{c,f}$

in both regimes. Differencing it across regimes,

$$\beta_b = \omega + \frac{\Delta \log f_{c,f}}{\Delta \log n_f} + \frac{\Delta \widetilde{\text{Const}}}{\Delta \log n_f}.$$

That is, β_b is ω, plus a composition effect in $f_{c,f}$, which is negative because adoption selects firms with low scope-maintenance costs, plus the level jump at adoption. No term carries $\frac{\rho\alpha\eta}{1-\eta}$ as an elasticity, as β_1 does, so the between-regime variation is informative about ω rather than about scope economies.

Substituting $\beta_b \approx \omega$ into equation (B.3),

$$\beta \approx \frac{\rho\alpha\eta}{1-\eta}\,\vartheta\hat{d}\left(1 - R^2_{z,1}\right) + \omega\left[\vartheta\hat{d}R^2_{z,1} + \vartheta\left(1-\hat{d}\right)R^2_{z,0} + 1 - \vartheta\right].$$

The loading on scope economies is a product of three attenuation factors: ϑ, the within-regime share of scope dispersion; $\hat{d}$, the adopters' share of it; and $1 - R^2_{z,1}$, the share of adopters' dispersion generated by $f_{c,f}$ rather than by z_f. Everything else loads on ω. Identification therefore requires variation in $f_{c,f}$ within an adoption regime.

B.3.2. A Multi-Sector Extension

This subsection embeds the identification strategy of Section 5.2 in the model. I show that, at $\delta = 0$, a shock to the cost of materials in some of a firm's sectors is equivalent to a shock to its scope cost $f_{c,f}$. I then show that the instrument B_f recovers a weighted average of the firm-level elasticity of revenue per product to the number of products.

Setup. Firm f is active in K_f sectors, indexed by k, with $n_{k,f}$ products in sector k and $n_f = \sum_k n_{k,f}$. Scope costs are sector-specific,

$$\sum_k f_{c,f}\frac{n_{k,f}^{1+\omega}}{1+\omega}.$$

Product i in sector k has productivity $z_{i,f} = z_f(1+i)^{-\delta}$, produces $y_{i,k,f} = z_{i,f}\,\ell_{i,k,f}^{1-\alpha-\gamma}x_{i,k,f}^{\gamma}m_{i,k,f}^{\alpha}$, and pays P/a_k per unit of $x_{i,k,f}$. In terms of $\tilde{x}_{i,k,f} \equiv x_{i,k,f}/a_k$, which costs P,

$$y_{i,k,f} = z_{i,f}\,a_k^{\gamma}\,\ell_{i,k,f}^{1-\alpha-\gamma}\tilde{x}_{i,k,f}^{\gamma}m_{i,k,f}^{\alpha}.$$

A change in a_k is therefore a Hicks-neutral change in the product's productivity, from $z_{i,f}$ to $z_{i,f}a_k^{\gamma}$, in both adoption regimes: the managerial input enters separately whether or not it is non-rival. In logs the shock is additive with the other components of productivity,

$$\log\left(z_{i,f}\,a_k^{\gamma}\right) = \log z_f + \gamma\log a_k - \delta\log(1+i).$$

On the core-competency ladder, i is the product's rank within the firm, not within its sector. I assume that at every rung each sector the firm is active in holds the same share of products, so no sector sits at the top or bottom of the ladder. This assumption matters only when $\delta > 0$.

Sectoral Shocks. The economy starts in a steady state with $a_k = 1$ for all k, so sectors are symmetric and each of the firm's sectors accounts for a share $1/K_f$ of its products. Following the empirical design, let G be the set of exposed sectors, and let $s_f \equiv \sum_{k \in G} s_{f,k}$ be the share of the firm's sectors that are exposed. All exposed sectors receive the same adverse shock, $a_k = a_E < 1$, a rise in their materials cost; the rest keep $a_k = 1$. I assume the shock does not change the adoption decision d_f, and I study it in partial equilibrium, holding A, W, and P fixed, consistent with the year fixed effect in the empirical specification. I compare allocations before and after the shock.

Mapping to the Data. Intermediate expenditure is a share γ of variable cost, and variable cost is a share η of revenue, so the intermediate share of output is $\gamma\eta$. The cost shock in the data, measured with input shares relative to output, is $\text{costshock}_k = -\gamma\eta \log a_k$, positive when materials become more expensive. With a share s_f of sectors exposed to a_E, the instrument in the model is

$$B_f = \sum_{k \in G} s_{f,k} \, \text{costshock}_k = -s_f \, \gamma\eta \log a_E.$$

For identification, I assume the exposed share s_f is independent of z_f and $f_{c,f}$. Since d_f and pre-shock scope are functions of $(z_f, f_{c,f})$, s_f is independent of them too.

Effective Scope. Define effective scope as the firm's products weighted by their productivity relative to z_f, including the cost shock:

$$\Sigma_f \equiv \int_{\text{products of } f} \left(\frac{z_{i,f} \, a_k^{\gamma}}{z_f} \right)^{\frac{\eta}{1-\eta(1-\rho\alpha d_f)}} di.$$

Scope enters production only through Σ_f: $Z_f = z_f \, \Sigma_f^{\frac{1-\eta(1-\rho\alpha d_f)}{\eta}}$. Since $S_f \propto Z_f^{\frac{\eta}{1-\eta}}$ and variable costs are a share η of sales,

$$\Pi_f = \text{Const}_f \, \Sigma_f^{1+\frac{\rho\alpha\eta}{1-\eta} d_f} - \text{scope cost}, \qquad \text{Const}_f \propto z_f^{\frac{\eta}{1-\eta}}.$$

With no shock, the firm's n_f products occupy rungs $[0, n_f]$ at density one, so $\Sigma_f = \mathcal{A}(n_f) \equiv \int_0^{n_f} (1+i)^{-\delta \frac{\eta}{1-\eta(1-\rho\alpha d_f)}} di$, and $\mathcal{A}(n_f) = n_f$ when $\delta = 0$.

Baseline Model. With no shock and $K_f = 1$, $\Sigma_f = \mathcal{A}(n_f)$ and Π_f is the baseline profit. With no shock and K_f sectors, the scope cost is $K_f f_{c,f} \frac{(n_f/K_f)^{1+\omega}}{1+\omega} = f_{c,f} K_f^{-\omega} \frac{n_f^{1+\omega}}{1+\omega}$: the baseline

firm with scope cost $f_{c,f}K_f^{-\omega}$. The calibrated distribution of the scope cost is the distribution of $f_{c,f}K_f^{-\omega}$, so the calibrated economy is the pre-shock multi-sector economy.

Equivalence between Cost Shocks ($\delta = 0$). At $\delta = 0$ every product in sector k has weight $a_k^{\gamma\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$ in Σ_f, so $\Sigma_f = \sum_k n_{k,f}\, a_k^{\gamma\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$. The scope FOC for sector k is

$$f_{c,f}\, n_{k,f}^{\omega} = \text{Const}_f\; \Sigma_f^{\frac{\rho\alpha\eta}{1-\eta}d_f}\; a_k^{\gamma\frac{\eta}{1-\eta(1-\rho\alpha d_f)}},$$

with Const_f now including the factor $1 + \frac{\rho\alpha\eta}{1-\eta}d_f$. Solving for each sector,

$$n_{k,f} = \left(\frac{\text{Const}_f\; \Sigma_f^{\frac{\rho\alpha\eta}{1-\eta}d_f}}{f_{c,f}}\right)^{\frac{1}{\omega}} a_k^{\frac{\gamma}{\omega}\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}.$$

The only sector-specific term is a_k. Unexposed sectors therefore all have the same scope, $n_{U,f}$, and exposed sectors all have $n_{E,f}$, with

$$\mu_f \equiv \frac{n_{E,f}}{n_{U,f}} = a_E^{\frac{\gamma}{\omega}\frac{\eta}{1-\eta(1-\rho\alpha d_f)}} \in (0,1).$$

Multiplying the per-sector solution by $a_k^{\gamma\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$ and summing over k, the left side is Σ_f:

$$\Sigma_f = \left(\frac{\text{Const}_f\; \Sigma_f^{\frac{\rho\alpha\eta}{1-\eta}d_f}}{f_{c,f}}\right)^{\frac{1}{\omega}} Q_f, \qquad Q_f \equiv \sum_k a_k^{\gamma\frac{1+\omega}{\omega}\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}.$$

Rearranging,

$$\left(\omega - \frac{\rho\alpha\eta}{1-\eta}d_f\right)\log\Sigma_f = \log\text{Const}_f - \log\left(f_{c,f}\, Q_f^{-\omega}\right).$$

This is the baseline scope FOC with Σ_f in place of n_f and the effective scope cost $f_{c,f}Q_f^{-\omega}$ in place of the scope cost. Q_f counts the sectors the firm is active in, each weighted by how cheap it is to produce in; with no shock, $Q_f = K_f$. The shocks to a_k enter only through Q_f, and hence enter the FOC for effective scope exactly as $f_{c,f}$ does.

Responses. The regression is in terms of n_f rather than Σ_f. Summing the per-sector solution without the weights, and using the previous display to replace the common factor by Σ_f/Q_f,

$$n_f = \frac{\Sigma_f}{Q_f}\sum_k a_k^{\frac{\gamma}{\omega}\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}.$$

With no shock, $n_f = \Sigma_f$. With a share s_f of sectors exposed,

$$Q_f = K_f \left[(1 - s_f) + s_f \, \mu_f^{1+\omega}\right], \qquad \sum_k a_k^{\frac{\gamma}{\omega}\frac{\eta}{1-\eta(1-\rho\alpha d_f)}} = K_f \left[(1 - s_f) + s_f \, \mu_f\right],$$

so after the shock

$$n_f = \Sigma_f \, \frac{1 - s_f + s_f \, \mu_f}{1 - s_f + s_f \, \mu_f^{1+\omega}}.$$

Let X^0 denote the value of X before the shock and $\Delta X \equiv X - X^0$. Let r_f be the average log revenue of the firm's unexposed products. At $\delta = 0$, every unexposed product has the same revenue, and

$$r_f = \text{Const} + \frac{\rho\alpha\eta}{1-\eta} d_f \log \Sigma_f + \frac{\eta}{1-\eta} \log z_f.$$

Since $Q_f^0 = K_f$ and the constants are fixed in partial equilibrium,

$$\begin{aligned}
\Delta \log \Sigma_f &= \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f} \log \left[1 - s_f + s_f \, \mu_f^{1+\omega}\right], \\
\Delta r_f &= \frac{\rho\alpha\eta}{1-\eta} d_f \, \Delta \log \Sigma_f, \\
\Delta \log n_f &= \Delta \log \Sigma_f + \log \frac{1 - s_f + s_f \, \mu_f}{1 - s_f + s_f \, \mu_f^{1+\omega}}.
\end{aligned}$$

If a_E is small enough that exposed products exit, $\mu_f \to 0$, then

$$\begin{aligned}
\Delta r_f &= \frac{\rho\alpha\eta}{1-\eta} d_f \, \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f} \log(1 - s_f), \\
\Delta \log n_f &= \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f} \log(1 - s_f),
\end{aligned}$$

and

$$\beta_f \equiv \frac{\Delta r_f}{\Delta \log n_f} = \frac{\rho\alpha\eta}{1-\eta} d_f.$$

Within a firm, the response of the average revenue of unexposed products relative to the response of scope recovers the firm's elasticity of interest.

The IV Estimand. The regression has one observation per firm and year, with the average log revenue of unexposed products, r_f, as the dependent variable:

$$\beta_{IV} = \frac{\mathbb{C}ov\left(r_f, B_f\right)}{\mathbb{C}ov\left(\log n_f, B_f\right)}.$$

From the responses above,

$$r_f = r_f^0 + \frac{\rho\alpha\eta}{1-\eta} d_f \, \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f} \log(1 - s_f),$$
$$\log n_f = \log n_f^0 + \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f} \log(1 - s_f).$$

The pre-shock values r_f^0 and $\log n_f^0$ and the adoption decision d_f are functions of $(z_f, f_{c,f})$, so they are independent of s_f. With $B_f = -s_f \gamma\eta \log a_E$,

$$\mathbb{C}ov\left(\log n_f, B_f\right) = \mathbb{E}\left[\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}\right] \mathbb{C}ov\left(\log(1 - s_f), s_f\right) \left(-\gamma\eta \log a_E\right),$$
$$\mathbb{C}ov\left(r_f, B_f\right) = \mathbb{E}\left[\frac{\rho\alpha\eta}{1-\eta} d_f \, \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}\right] \mathbb{C}ov\left(\log(1 - s_f), s_f\right) \left(-\gamma\eta \log a_E\right).$$

The common factor cancels. With $\bar{d} \equiv \mathbb{E}[d_f]$ the adoption rate,

$$\beta_{IV} = \frac{\rho\alpha\eta}{1-\eta} \bar{d} \cdot \frac{\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}}{\bar{d} \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}} + 1 - \bar{d}} = \frac{\rho\alpha\eta}{1-\eta} \frac{\bar{d}\,\omega}{\omega - (1 - \bar{d}) \frac{\rho\alpha\eta}{1-\eta}}.$$

β_{IV} is therefore $\frac{\rho\alpha\eta}{1-\eta}\bar{d}$ scaled by the ratio of the adopters' first stage to the average first stage. That ratio exceeds one, because adopters' scope responds more to the shock.

Proposition 8. *Assume $\delta = 0$; the shock does not change d_f; the exposed share s_f is independent of z_f and $f_{c,f}$; all exposed sectors receive the same shock $a_E < 1$; and a_E is small enough that exposed products exit, $n_{E,f}/n_{U,f} \to 0$. Then, in partial equilibrium, the population IV coefficient is*

$$\beta_{IV} = \frac{\rho\alpha\eta}{1-\eta} \bar{d} \cdot \frac{\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}}{\bar{d} \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}} + 1 - \bar{d}}, \qquad \bar{d} = \mathbb{E}[d_f].$$

Computing β_{IV} in the Single-Sector Model. Under the assumptions of the proposition, $Q_f = K_f(1 - s_f)$, so the shock raises the effective scope cost by a factor common to all firms:

$$\log\left(f_{c,f} Q_f^{-\omega}\right) = \log\left(f_{c,f} K_f^{-\omega}\right) - \omega \log(1 - s_f),$$

and $n_f = \Sigma_f$. Let n_f^{ss} and r_f^{ss} be scope and average log revenue in the single-sector model with scope cost $f_{c,f} K_f^{-\omega}$. In that model,

$$\frac{\partial \log n_f^{ss}}{\partial \log f_{c,f}} = -\frac{1}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}, \qquad \frac{\partial r_f^{ss}}{\partial \log f_{c,f}} = -\frac{\frac{\rho\alpha\eta}{1-\eta} d_f}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}.$$

By the chain rule, the responses of the multi-sector model to the shock are these derivatives times $-\omega$:

$$\frac{\partial \log n_f}{\partial \log(1-s_f)} = \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}, \qquad \frac{\partial r_f}{\partial \log(1-s_f)} = \frac{\rho\alpha\eta}{1-\eta} d_f \frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta} d_f}.$$

The averages of these responses are the first stage and the reduced form of the IV, and the common factor $-\omega$ cancels in their ratio.

Proposition 9. *Under the assumptions of Proposition 8, the multi-sector β_{IV} can be computed from the single-sector model's response to its scope cost:*

$$\beta_{IV} = \frac{\mathbb{E}\left[\frac{\partial r_f^{ss}}{\partial \log f_{c,f}}\right]}{\mathbb{E}\left[\frac{\partial \log n_f^{ss}}{\partial \log f_{c,f}}\right]}.$$

Core Competency ($\delta > 0$). The setup and assumptions are those of Proposition 8, except that $\delta > 0$, with one addition on the ladder: a product's rank on the ladder does not change when other products leave, so the remaining products keep their productivity. Together with the assumption that sectors are interleaved on the ladder, this determines the ranks of the products that remain after the shock.

Exit. Adding an exposed product raises Σ_f by at most its own weight, which is at most $a_E^{\gamma \frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$, since the ladder term $(1+i)^{-\delta \frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$ is at most one and other products keep their ranks. The scope FOC for an exposed sector therefore gives

$$f_{c,f}\, n_{E,f}^{\omega} \;\le\; \text{Const}_f\; \Sigma_f^{\frac{\rho\alpha\eta}{1-\eta} d_f}\; a_E^{\gamma \frac{\eta}{1-\eta(1-\rho\alpha d_f)}}.$$

As $a_E \to 0$, the right side goes to zero and $n_{E,f} \to 0$: exposed sectors exit, and the firm is left with $(1-s_f)K_f$ sectors. Unlike at $\delta = 0$, this step needs exit: for intermediate a_E, the ranks of exposed and unexposed products depend on $n_{E,f}/n_{U,f}$.

Effective Scope after the Shock. Before the shock, the firm's n_f products are interleaved on ranks $[0, n_f]$, so $\Sigma_f = \mathcal{A}(n_f)$, and the rank of the firm's lowest-productivity product is n_f. Exit removes a share s_f of products at every rank, and the remaining products keep their ranks. After the shock, the firm's n_f products therefore occupy ranks $[0, m_f]$ at density $1 - s_f$, where

$$m_f \equiv \frac{n_f}{1-s_f}$$

is the rank of the firm's lowest-productivity product. Each additional product raises this rank by $1/(1-s_f)$, because the exposed sectors' products no longer occupy the ranks in between.

Effective scope is

$$\Sigma_f = (1 - s_f)\,\mathcal{A}(m_f).$$

The Firm's Problem after the Shock. $(1 - s_f)K_f$ sectors remain, with $n_f/((1 - s_f)K_f)$ products each, so the scope cost is

$$\begin{aligned}(1 - s_f)K_f\, f_{c,f}\frac{\left(\frac{n_f}{(1-s_f)K_f}\right)^{1+\omega}}{1+\omega} &= f_{c,f}K_f^{-\omega}(1 - s_f)^{-\omega}\frac{n_f^{1+\omega}}{1+\omega}\\ &= f_{c,f}K_f^{-\omega}(1 - s_f)\frac{m_f^{1+\omega}}{1+\omega},\end{aligned}$$

using $n_f = (1 - s_f)m_f$. Profit, as a function of m_f, is

$$\Pi_f = \text{Const}_f\,[(1 - s_f)\,\mathcal{A}(m_f)]^{1+\frac{\rho\alpha\eta}{1-\eta}d_f} - f_{c,f}K_f^{-\omega}(1 - s_f)\frac{m_f^{1+\omega}}{1+\omega}.$$

The firm chooses n_f; for given s_f, $m_f = n_f/(1 - s_f)$ is a one-to-one transformation of n_f, so the FOC can equivalently be written in m_f. I write it in m_f because the productivity of the marginal product depends on its rank, so the ladder terms take the same form as in the baseline model and the shock enters only through the scope cost. The FOC is

$$\text{Const}_f\;\mathcal{A}(m_f)^{\frac{\rho\alpha\eta}{1-\eta}d_f}\,\mathcal{A}'(m_f) = f_{c,f}K_f^{-\omega}(1 - s_f)^{-\frac{\rho\alpha\eta}{1-\eta}d_f}\,m_f^{\omega},$$

with Const_f now including the factor $1 + \frac{\rho\alpha\eta}{1-\eta}d_f$ and $\mathcal{A}'(m) = (1 + m)^{-\delta\frac{\eta}{1-\eta(1-\rho\alpha d_f)}}$. For non-adopters the right side does not depend on s_f: m_f is unchanged and their number of products falls by exactly the factor $1-s_f$. For adopters, losing sectors raises the right side: fewer products share the non-rival input, so adding products is more costly at the margin, and they lower m_f.

Responses. The variables in the regression are the log number of products, $\log n_f = \log(1 - s_f) + \log m_f$, and the average log revenue of unexposed products. The remaining products have ranks uniformly distributed on $[0, m_f]$, so the average of $\log z_{i,f}$ among them is $\log z_f - \delta\bar{L}(m_f)$, with $\bar{L}(m) = \frac{(1+m)\log(1+m)-m}{m}$ the mean of $\log(1 + i)$ on $[0, m]$. Hence

$$r_f = \text{Const} + \frac{\rho\alpha\eta}{1-\eta}d_f\,[\log(1 - s_f) + \log\mathcal{A}(m_f)] + \frac{\eta}{1-\eta}\log z_f - \frac{\eta\,\delta}{1-\eta(1-\rho\alpha d_f)}\bar{L}(m_f).$$

The last term is the core-competency composition effect: lowering m_f drops the lowest-productivity products and raises the average revenue of the remaining ones.

At $s_f = 0$, $m_f = n_f$. Let $\epsilon_f^{\mathcal{A}} \equiv \frac{n_f\mathcal{A}'(n_f)}{\mathcal{A}(n_f)} \in (0, 1]$ and $\epsilon_f^{L} \equiv n_f\bar{L}'(n_f) = \frac{n_f-\log(1+n_f)}{n_f} \in (0, 1)$ be the elasticities of effective scope and of the composition term with respect to m_f, and

$$\kappa_f \equiv \omega + \frac{\eta\,\delta}{1-\eta(1-\rho\alpha d_f)}\frac{n_f}{1+n_f} - \frac{\rho\alpha\eta}{1-\eta}d_f\,\epsilon_f^{\mathcal{A}} > 0$$

the curvature of the FOC for m_f. Taking logs of the FOC and differentiating with respect to $\log(1-s_f)$,

$$\frac{\partial \log m_f}{\partial \log(1-s_f)} = \frac{\frac{\rho\alpha\eta}{1-\eta} d_f}{\kappa_f}.$$

The responses of the number of products and of revenue are therefore

$$\pi_f \equiv \frac{\partial \log n_f}{\partial \log(1-s_f)} = 1 + \frac{\frac{\rho\alpha\eta}{1-\eta} d_f}{\kappa_f},$$
$$b_f \equiv \frac{\partial r_f}{\partial \log(1-s_f)} = \frac{\rho\alpha\eta}{1-\eta} d_f \left(1 + \epsilon_f^{\mathcal{A}} \frac{\frac{\rho\alpha\eta}{1-\eta} d_f}{\kappa_f}\right) - \frac{\eta\,\delta}{1-\eta(1-\rho\alpha d_f)} \epsilon_f^L \frac{\frac{\rho\alpha\eta}{1-\eta} d_f}{\kappa_f}.$$

Non-adopters have $\pi_f = 1$ and $b_f = 0$. For adopters,

$$\beta_f \equiv \frac{b_f}{\pi_f} = \frac{\rho\alpha\eta}{1-\eta} \frac{\omega + \frac{\eta\,\delta}{1-\eta(1-\rho\alpha)} \left(\frac{n_f}{1+n_f} - \epsilon_f^L\right)}{\omega + \frac{\eta\,\delta}{1-\eta(1-\rho\alpha)} \frac{n_f}{1+n_f} + \frac{\rho\alpha\eta}{1-\eta}\left(1 - \epsilon_f^{\mathcal{A}}\right)} < \frac{\rho\alpha\eta}{1-\eta}.$$

Two forces pull the adopter's elasticity below $\frac{\rho\alpha\eta}{1-\eta}$: the products dropped are the lowest-productivity ones, so effective scope falls by less than the number of products ($\epsilon_f^{\mathcal{A}} < 1$), and dropping them raises the average revenue of the remaining products ($\epsilon_f^L > 0$). At $\delta = 0$, $\epsilon_f^{\mathcal{A}} = 1$, the composition term drops out, $\kappa_f = \omega - \frac{\rho\alpha\eta}{1-\eta} d_f$, and $\beta_f = \frac{\rho\alpha\eta}{1-\eta} d_f$.

Core competency also lowers the first stage. Since $\frac{\eta\,\delta}{1-\eta(1-\rho\alpha d_f)} \frac{n_f}{1+n_f} > 0$ and $\epsilon_f^{\mathcal{A}} \leq 1$, $\kappa_f > \omega - \frac{\rho\alpha\eta}{1-\eta} d_f$, so an adopter's first stage, $\pi_f = 1 + \frac{\rho\alpha\eta}{1-\eta}/\kappa_f$, is below its value at $\delta = 0$, $\frac{\omega}{\omega - \frac{\rho\alpha\eta}{1-\eta}}$. Non-adopters' first stage is one in both cases. The ratio of adopters' first stage to the average first stage, which scales β_{IV} in Proposition 8, is therefore closer to one.

The IV Estimand. The outcomes are no longer exactly linear in $\log(1-s_f)$, so the result is first order in s_f. With $\log(1-s_f) = -s_f + O(s_f^2)$,

$$r_f = r_f^0 - b_f\, s_f, \qquad \log n_f = \log n_f^0 - \pi_f\, s_f.$$

The pre-shock values, d_f, b_f, and π_f are functions of $(z_f, f_{c,f})$ and hence independent of s_f. With $B_f = -s_f \gamma\eta \log a_E$,

$$\mathbb{C}ov(r_f, B_f) = \gamma\eta \log a_E\, \mathbb{V}ar(s_f)\, \mathbb{E}[b_f], \qquad \mathbb{C}ov(\log n_f, B_f) = \gamma\eta \log a_E\, \mathbb{V}ar(s_f)\, \mathbb{E}[\pi_f],$$

and the common factor cancels.

Proposition 10. *Under the assumptions of Proposition 8, but with $\delta > 0$ and remaining products keeping their ranks, to first order in s_f the population IV coefficient is*

$$\beta_{IV} = \frac{\mathbb{E}[b_f]}{\mathbb{E}[\pi_f]},$$

with b_f and π_f as above, evaluated at each firm's pre-shock scope n_f.

Computing β_{IV} in the Single-Sector Model. The FOC for m_f is the single-sector scope FOC, with m_f in place of n_f and the scope cost raised from $f_{c,f}K_f^{-\omega}$ to

$$f_{c,f}K_f^{-\omega}\left(1-s_f\right)^{-\frac{\rho\alpha\eta}{1-\eta}d_f}.$$

With n_f^{ss} and r_f^{ss} defined as at $\delta = 0$, in the single-sector model

$$\frac{\partial \log n_f^{ss}}{\partial \log f_{c,f}} = -\frac{1}{\kappa_f}, \qquad \frac{\partial r_f^{ss}}{\partial \log f_{c,f}} = -\frac{\frac{\rho\alpha\eta}{1-\eta}d_f\,\epsilon_f^{\mathcal{A}} - \frac{\eta\,\delta}{1-\eta(1-\rho\alpha d_f)}\epsilon_f^{L}}{\kappa_f}.$$

Since $\log n_f = \log(1-s_f) + \log m_f$ and $r_f = \frac{\rho\alpha\eta}{1-\eta}d_f \log(1-s_f) + r_f^{ss}(m_f)$, the chain rule gives

$$\pi_f = 1 - \frac{\rho\alpha\eta}{1-\eta}d_f\,\frac{\partial \log n_f^{ss}}{\partial \log f_{c,f}}, \qquad b_f = \frac{\rho\alpha\eta}{1-\eta}d_f\left(1 - \frac{\partial r_f^{ss}}{\partial \log f_{c,f}}\right).$$

Proposition 11. *Under the assumptions of Proposition 10, the multi-sector β_{IV} can be computed from the single-sector model's response to its scope cost:*

$$\beta_{IV} = \frac{\frac{\rho\alpha\eta}{1-\eta}\,\mathbb{E}\left[d_f\left(1 - \frac{\partial r_f^{ss}}{\partial \log f_{c,f}}\right)\right]}{\mathbb{E}\left[1 - \frac{\rho\alpha\eta}{1-\eta}d_f\,\frac{\partial \log n_f^{ss}}{\partial \log f_{c,f}}\right]}.$$

Unlike at $\delta = 0$, the shift in the scope cost, $(1-s_f)^{-\frac{\rho\alpha\eta}{1-\eta}d_f}$, differs between adopters and non-adopters, so the factor relating the shock to the scope cost does not cancel and stays inside the expectations.

B.4. Aggregation and Amplification

B.4.1. Product-Level Distortions

Section 6 introduces a firm-level wedge $\tau(z_f)$. Here I allow wedges to vary across products within the firm. Let $\tau_{i,f}$ be the revenue wedge on product i of firm f, for instance $\tau_{i,f} = \tau_0 z_{i,f}^{-\xi}$, which taxes more productive products more heavily. Product-level wedges are relevant because some policies, such as the product reservations in India studied in Table 3, target specific products and sectors rather than firms. The firm-level wedge is the special case $\tau_{i,f} = \tau(z_f)$ for all i. I state the derivations for $d_f = 1$; the case $d_f = 0$ is the same with $\rho = 0$ and Ω_0 in place of Ω_1.

Firm-Level Productivity. The firm receives $\tau_{i,f} p_{i,f} y_{i,f}$ from product i, in place of $p_{i,f} y_{i,f}$ in problem (8). Substituting the demand (2) and the technology (10),

$$\tau_{i,f} p_{i,f} y_{i,f} = A \left(\tau_{i,f}^{\frac{1}{\eta}} z_{i,f} \ell_{i,f}^{1-\alpha-\gamma} x_{i,f}^{\gamma} m_{i,f}^{\alpha} \right)^{\eta},$$

so the wedge acts as a shifter of product productivity: the problem is the undistorted one with $\tau_{i,f}^{1/\eta} z_{i,f}$ in place of $z_{i,f}$. The proof of Proposition 1 goes through with this replacement, and profits take the form in Section 6 with firm-level productivity

$$\tilde{Z}_f = \left(\int_0^{n_f} \left(\tau_{i,f}^{\frac{1}{\eta}} z_{i,f} \right)^{\frac{\eta}{1-\eta(1-\rho\alpha)}} di \right)^{\frac{1-\eta(1-\rho\alpha)}{\eta}},$$

which equals $\tau(z_f)^{1/\eta} Z_f$ when wedges are firm-level. Within-firm allocations are as in equation (15) with $\tau_{i,f}^{1/\eta} z_{i,f}$ and $\tilde{Z}_f$ in place of $z_{i,f}$ and Z_f, and firm-level factor demands are as in Section 4.1 with $\tilde{Z}_f$ in place of Z_f. In particular, the cost share of product i, which equals its share in the revenue the firm receives, is

$$c_{i,f} \equiv \frac{\ell_{i,f}}{\ell_f} = \frac{\tau_{i,f} p_{i,f} y_{i,f}}{\int_0^{n_f} \tau_{j,f} p_{j,f} y_{j,f} \, dj} = \left(\frac{\tau_{i,f}^{\frac{1}{\eta}} z_{i,f}}{\tilde{Z}_f} \right)^{\frac{\eta}{1-\eta(1-\rho\alpha)}},$$

and the revenue the firm receives is $\int_0^{n_f} \tau_{i,f} p_{i,f} y_{i,f} \, di = A \left(A\eta / \Omega_1 \right)^{\frac{\eta}{1-\eta}} \tilde{Z}_f^{\frac{\eta}{1-\eta}}$.

Within-Firm Misallocation. Define the firm's average wedge as the ratio of the revenue it receives to its sales. Since $p_{i,f} y_{i,f} = \tau_{i,f}^{-1} \tau_{i,f} p_{i,f} y_{i,f}$, it is the harmonic cost-weighted average of the firm's product-level wedges,

$$\bar{\tau}_f \equiv \frac{\int_0^{n_f} \tau_{i,f} p_{i,f} y_{i,f} \, di}{\int_0^{n_f} p_{i,f} y_{i,f} \, di}, \qquad \frac{1}{\bar{\tau}_f} = \int_0^{n_f} \frac{1}{\tau_{i,f}} c_{i,f} \, di,$$

which equals $\tau(z_f)$ when wedges are firm-level. Unlike with firm-level wedges, sales shares within the firm differ from cost shares: the sales share of product i is $\frac{\bar{\tau}_f}{\tau_{i,f}} c_{i,f}$.

To compare $\tilde{Z}_f$ with its firm-level counterpart $\tau(z_f)^{1/\eta} Z_f$, write it in terms of Z_f, the undistorted firm-level productivity in Proposition 1 evaluated at the firm's choices of n_f and d_f. Let $\nu \equiv \frac{1}{1-\eta(1-\rho\alpha)}$, so that the exponent in Proposition 1 is $\nu\eta$, and let

$$\varpi_{i,f} \equiv \left(\frac{z_{i,f}}{Z_f} \right)^{\nu\eta}$$

be the cost shares the firm would choose at the same n_f and d_f absent wedges. Then

$$\tilde{Z}_f = \bar{\tau}_f^{\frac{1}{\eta}} \, \mathcal{U}_f \, Z_f, \qquad \mathcal{U}_f \equiv \left(\int_0^{n_f} \left(\frac{\tau_{i,f}}{\bar{\tau}_f} \right)^{\nu} \varpi_{i,f} \, di \right)^{\frac{1}{\nu\eta}}.$$

Relative to the firm-level case, the average wedge $\bar{\tau}_f$ replaces $\tau(z_f)$, and $\mathcal{U}_f$ captures misallocation within the firm, due to dispersion in wedges across its products. If wedges are constant within the firm, then $\tau_{i,f} = \bar{\tau}_f$ and $\mathcal{U}_f = 1$.

Within-Firm Dispersion Lowers Productivity. I show that $\mathcal{U}_f \leq 1$, with equality if and only if $\tau_{i,f}$ is constant across the firm's products. Let

$$M_r \equiv \left(\int_0^{n_f} \varpi_{i,f} \tau_{i,f}^r \, di \right)^{\frac{1}{r}}$$

be the power mean of order r of the firm's wedges under the weights $\varpi_{i,f}$. Since $\left(\tau_{i,f}^{1/\eta} z_{i,f} \right)^{\nu\eta} = \tau_{i,f}^{\nu} z_{i,f}^{\nu\eta}$, cost shares are $c_{i,f} = \varpi_{i,f} \tau_{i,f}^{\nu} / M_\nu^\nu$, so

$$\frac{1}{\bar{\tau}_f} = \frac{M_{\nu-1}^{\nu-1}}{M_\nu^\nu}, \qquad \mathcal{U}_f^\eta = \frac{M_\nu}{\bar{\tau}_f} = \left(\frac{M_{\nu-1}}{M_\nu} \right)^{\nu-1}.$$

Since $0 < \eta(1 - \rho\alpha) < 1$, $\nu > 1$, so $0 < \nu - 1 < \nu$. By the power mean inequality, $M_{\nu-1} \leq M_\nu$, with equality if and only if $\tau_{i,f}$ is constant across products. Hence $\mathcal{U}_f \leq 1$. Holding n_f and d_f fixed, dispersion in wedges within the firm therefore lowers $\tilde{Z}_f$ below its value under a uniform wedge equal to $\bar{\tau}_f$. The two weightings are what deliver this result: $\bar{\tau}_f$ weights wedges by the distorted cost shares $c_{i,f}$, while $\mathcal{U}_f$ weights them by the undistorted shares $\varpi_{i,f}$.

B.4.2. Aggregation

This subsection derives the expressions in Section 6.1, with firm-level wedges $\tau_f \equiv \tau(z_f)$. As there, I set $\phi = 0$, so that $P_m^1 = P_m^0 = W$, $\Omega_0 = \Omega_1 = \Omega$, and the managerial input is labor for every firm, $m_f = \ell_f^m$. I use $P = 1$, so that $A = Y^{\frac{1}{\sigma}} = Y^{1-\eta}$. Let $S_f = \int_0^{n_f} p_{i,f} y_{i,f} \, di$ be the sales of firm f. Throughout, I assume aggregate sales are finite; without distortions, this requires the tail index of sales to exceed one, $\zeta_n^{(1)} > 1 + \omega$ (Proposition 4), and with distortions I assume the same of the integrals below.

Firm Allocations. By Section 6, firm f solves the undistorted problem with $\tilde{Z}_f = \tau_f^{1/\eta} Z_f$ in place of Z_f. Its factor demands are therefore those in Section 4.1 with $\tilde{Z}_f$ in place of Z_f,

$$\ell_f = (A\eta/\Omega)^{\frac{1}{1-\eta}} \, \Omega \frac{1 - \alpha - \gamma}{W} \tilde{Z}_f^{\frac{\eta}{1-\eta}}, \qquad x_f = (A\eta/\Omega)^{\frac{1}{1-\eta}} \, \Omega \, \gamma \, \tilde{Z}_f^{\frac{\eta}{1-\eta}}, \qquad m_f = (A\eta/\Omega)^{\frac{1}{1-\eta}} \, \Omega \frac{\alpha}{W} \tilde{Z}_f^{\frac{\eta}{1-\eta}},$$

and the revenue it receives is $\tau_f S_f = A\left(A\eta/\Omega\right)^{\frac{\eta}{1-\eta}} \tilde{Z}_f^{\frac{\eta}{1-\eta}}$, of which a share η is paid to variable inputs.

Price Equation. With $P = 1$, $Y = \int S_f \, df = \int \tau_f^{-1} \tau_f S_f \, df$. Substituting the revenue the firm receives and using $A^{\frac{1}{1-\eta}} = Y$,

$$Y = A^{\frac{1}{1-\eta}} \left(\frac{\eta}{\Omega}\right)^{\frac{\eta}{1-\eta}} \int \frac{1}{\tau_f} \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df \qquad \Longrightarrow \qquad \left(\frac{\Omega}{\eta}\right)^{\frac{\eta}{1-\eta}} = \int \frac{1}{\tau_f} \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df.$$

With $\bar{\tau}$ as defined in Section 6.1, the right-hand side equals $\bar{\tau}^{-1} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df$. Since $\tilde{Z}_f^{\frac{\eta}{1-\eta}} = \tau_f^{\frac{1}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}}$, aggregate productivity as defined in Section 6.1 satisfies

$$Z^{\frac{\eta}{1-\eta}} = \int \left(\frac{\tau_f}{\bar{\tau}}\right)^{\frac{1}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} \, df = \bar{\tau}^{-\frac{1}{1-\eta}} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df,$$

and the price equation becomes

$$\Omega = \eta \bar{\tau} Z. \tag{B.4}$$

The ratio $\tau_f/\bar{\tau}$ isolates misallocation between firms. Under a uniform wedge $\tau_f = \tau_0$, $\bar{\tau} = \tau_0$ and $Z = \left(\int Z_f^{\frac{\eta}{1-\eta}} df\right)^{\frac{1-\eta}{\eta}}$, so the wedge affects Z only through the firms' choices of n_f and d_f.

Aggregate Productivity. Let $L^\ell = \int \ell_f \, df$, $L^m = \int \ell_f^m \, df = \int m_f \, df$, and $X = \int x_f \, df$ be aggregate manual labor, managerial labor, and materials. Aggregating the factor demands and using $\Omega = \left(\frac{W}{1-\alpha-\gamma}\right)^{1-\alpha-\gamma} \gamma^{-\gamma} \left(\frac{W}{\alpha}\right)^{\alpha}$,

$$\left(L^\ell\right)^{1-\alpha-\gamma} \left(L^m\right)^{\alpha} X^{\gamma} = \left(A\eta/\Omega\right)^{\frac{1}{1-\eta}} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df.$$

Dividing Y by this expression and using the price equation (B.4),

$$Y = Z \left(L^\ell\right)^{1-\alpha-\gamma} \left(L^m\right)^{\alpha} X^{\gamma},$$

so Z is aggregate total factor productivity for the product-level technology.

Factor Share. Since each firm pays a share η of the revenue it receives to variable inputs, aggregate variable costs are

$$W\left(L^\ell + L^m\right) + X = \eta \int \tau_f S_f \, df = \eta \bar{\tau} \, Y,$$

where the second equality follows from dividing $\int \tau_f S_f \, df = A\left(A\eta/\Omega\right)^{\frac{\eta}{1-\eta}} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df$ by the expression for Y above. The factor share is therefore $\eta\bar{\tau}$ and the aggregate markup is $\frac{1}{\eta\bar{\tau}}$. By

input, $W\left(L^{\ell}+L^{m}\right)=(1-\gamma)\eta\bar{\tau}Y$ and $X=\gamma\eta\bar{\tau}Y$.

Wages and Output. With $P=1$ and $P_m^d=W$, the unit cost is $\Omega=a_0W^{1-\gamma}$, where $a_0\equiv(1-\alpha-\gamma)^{-(1-\alpha-\gamma)}\gamma^{-\gamma}\alpha^{-\alpha}$. The price equation (B.4) then gives the wage,

$$W=\left(\frac{\eta}{a_0}\bar{\tau}Z\right)^{\frac{1}{1-\gamma}}.$$

Labor-market clearing, $L^{\ell}+L^{m}=1$, together with the labor share gives $W=(1-\gamma)\eta\bar{\tau}Y$, so

$$Y=\frac{1}{(1-\gamma)\eta}\left(\frac{\eta}{a_0}\right)^{\frac{1}{1-\gamma}}Z^{\frac{1}{1-\gamma}}\bar{\tau}^{\frac{\gamma}{1-\gamma}}, \tag{B.5}$$

which is the expression in Section 6.1. Because materials are produced from the final good, an increase in Z raises the wage, and hence output, more than proportionally. Final-good market clearing then gives consumption,

$$C=(1-\gamma\eta\bar{\tau})\,Y-\int\mathcal{C}_f(n_f)\,df-f_e\bar{d}.$$

Decomposition Weight. Component (3) of equation (38) is $\frac{1-\eta}{\eta}\big[\log(1+\bar{d}\,\mathcal{R}\,\mathcal{G})-\log(1+\bar{d}^{\star}\mathcal{R}^{\star}\mathcal{G}^{\star})\big]$, and the three pieces in equation (39) sum to $\log(\bar{d}\,\mathcal{R}\,\mathcal{G})-\log(\bar{d}^{\star}\mathcal{R}^{\star}\mathcal{G}^{\star})$. The weight that makes them sum exactly to component (3) is therefore

$$\bar{\mathcal{W}}=\frac{\log\left(1+\bar{d}\,\mathcal{R}\,\mathcal{G}\right)-\log\left(1+\bar{d}^{\star}\mathcal{R}^{\star}\mathcal{G}^{\star}\right)}{\log\left(\bar{d}\,\mathcal{R}\,\mathcal{G}\right)-\log\left(\bar{d}^{\star}\mathcal{R}^{\star}\mathcal{G}^{\star}\right)}.$$

B.4.3. Aggregation with Organization Capital

This subsection relaxes $\phi=0$. With $\phi>0$, adopters and non-adopters face different unit costs, $\Omega_1\neq\Omega_0$, and adopters use organization capital s_f^m alongside managerial labor. I keep $P=1$ and firm-level wedges τ_f, and follow the steps in Appendix B.4.2.

Cost-Adjusted Firm Productivity. The factor demands of firm f depend on its own unit cost Ω_{d_f}. Since $(A\eta/\Omega_d)^{\frac{1}{1-\eta}}\,\Omega_d=(A\eta/\Omega_0)^{\frac{1}{1-\eta}}\,\Omega_0\,(\Omega_0/\Omega_d)^{\frac{\eta}{1-\eta}}$, they can be written with the common unit cost Ω_0,

$$\ell_f=(A\eta/\Omega_0)^{\frac{1}{1-\eta}}\,\Omega_0\frac{1-\alpha-\gamma}{W}\hat{Z}_f^{\frac{\eta}{1-\eta}},\qquad x_f=(A\eta/\Omega_0)^{\frac{1}{1-\eta}}\,\Omega_0\,\gamma\,\hat{Z}_f^{\frac{\eta}{1-\eta}},\qquad m_f=(A\eta/\Omega_0)^{\frac{1}{1-\eta}}\,\Omega_0\frac{\alpha}{P_m^{d_f}}\hat{Z}_f^{\frac{\eta}{1-\eta}},$$

where

$$\hat{Z}_f\equiv\left(\frac{\Omega_0}{\Omega_1}\right)^{d_f}\tilde{Z}_f$$

is firm-level productivity adjusted for the cost advantage of adoption, $\frac{\Omega_0}{\Omega_1} = \left(\frac{W}{P_m^1}\right)^{\alpha}$. Likewise, the revenue the firm receives is $\tau_f S_f = A\left(A\eta/\Omega_0\right)^{\frac{\eta}{1-\eta}} \hat{Z}_f^{\frac{\eta}{1-\eta}}$. At $\phi = 0$, $\hat{Z}_f = \tilde{Z}_f$.

Price Equation and Factor Share. The same steps as in Appendix B.4.2 give

$$\left(\frac{\Omega_0}{\eta}\right)^{\frac{\eta}{1-\eta}} = \int \frac{1}{\tau_f} \hat{Z}_f^{\frac{\eta}{1-\eta}}\, df.$$

Define

$$\frac{1}{\bar{\tau}} = \int \frac{1}{\tau_f} \frac{\hat{Z}_f^{\frac{\eta}{1-\eta}}}{\int \hat{Z}_{f'}^{\frac{\eta}{1-\eta}}\, df'}\, df, \qquad Z = \left(\int \left(\frac{\tau_f}{\bar{\tau}}\right)^{\frac{1}{1-\eta}} \left(\left(\frac{\Omega_0}{\Omega_1}\right)^{d_f} Z_f\right)^{\frac{\eta}{1-\eta}} df\right)^{\frac{1-\eta}{\eta}},$$

so that $\Omega_0 = \eta\bar{\tau}Z$. These are the definitions in Section 6.1 with adopters' productivity scaled by the cost advantage. Because the weights in $\bar{\tau}$ now include $(\Omega_0/\Omega_1)^{\frac{\eta}{1-\eta}d_f}$, $\bar{\tau}$ differs from its value at $\phi = 0$ even for the same wedges $\tau(\cdot)$. Each firm still pays a share η of the revenue it receives to variable inputs, $W\ell_f + x_f + P_m^{d_f} m_f = \eta\tau_f S_f$, so the factor share is again $\eta\bar{\tau}$ and the aggregate markup is $\frac{1}{\eta\bar{\tau}}$. The unit cost of non-adopters is still $\Omega_0 = a_0 W^{1-\gamma}$, so the wage is

$$W = \left(\frac{\eta}{a_0}\bar{\tau}Z\right)^{\frac{1}{1-\gamma}},$$

as at $\phi = 0$.

Labor Market and Output. Non-adopters use managerial labor alone, $\ell_f^m = m_f$. Adopters pay a share $1-\phi$ of managerial costs to labor, $W\ell_f^m = (1-\phi)P_m^1 m_f$, and a share ϕ to organization capital, $s_f^m = \phi P_m^1 m_f$. Let

$$\chi \equiv \frac{\int d_f\, \tau_f S_f\, df}{\int \tau_f S_f\, df} = \frac{\int d_f \hat{Z}_f^{\frac{\eta}{1-\eta}}\, df}{\int \hat{Z}_f^{\frac{\eta}{1-\eta}}\, df}$$

be the share of adopters in input expenditure. Since wedges differ across firms, χ generally differs from adopters' share of sales. Using $W\ell_f = (1-\alpha-\gamma)\eta\tau_f S_f$ and $P_m^{d_f} m_f = \alpha\eta\tau_f S_f$, labor-market clearing gives

$$W = \int \left(W\ell_f + W\ell_f^m\right) df = \eta\left(1-\alpha-\gamma+\alpha(1-\chi)+\alpha(1-\phi)\chi\right)\int \tau_f S_f\, df = \eta\left(1-\gamma-\alpha\phi\chi\right)\bar{\tau}Y.$$

Substituting the wage,

$$Y = \frac{1}{(1-\gamma-\alpha\phi\chi)\eta}\left(\frac{\eta}{a_0}\right)^{\frac{1}{1-\gamma}} Z^{\frac{1}{1-\gamma}} \bar{\tau}^{\frac{\gamma}{1-\gamma}}.$$

This is equation (B.5) with $1-\gamma$ replaced by $1-\gamma-\alpha\phi\chi$ in the level term: organization capital is produced from the final good, so the labor share of costs falls by $\alpha\phi\chi$. The exponent on Z is unchanged, because the cost advantage enters through $\hat{Z}_f$ and hence through Z. Organization capital is $\int s_f^m \, df = \alpha\phi\chi\eta\bar{\tau}Y$, so final-good market clearing gives consumption,

$$C = \left(1 - \left(\gamma + \alpha\phi\chi\right)\eta\bar{\tau}\right) Y - \int \mathcal{C}_f(n_f)\, df - f_e\bar{d}.$$

Feedback Through the Cost Advantage. Since $P_m^1 \propto W^{1-\phi}$, the cost advantage rises with the wage, $\Omega_0/\Omega_1 \propto W^{\alpha\phi}$. A higher Z raises the wage, which raises the cost advantage and, through $\hat{Z}_f$, raises Z further. Holding Z_f, d_f, and $\bar{\tau}$ fixed, $\frac{\partial \log Z}{\partial \log(\Omega_0/\Omega_1)} = \chi$ and $\log W = \frac{1}{1-\gamma}\left(\log Z + \log\bar{\tau}\right) + \text{const}$, so this loop scales an initial change in $\log Z$ by

$$\left(1 - \frac{\alpha\phi\chi}{1-\gamma}\right)^{-1},$$

which is finite since $\alpha\phi\chi \leq \alpha < 1-\gamma$.

Aggregate Productivity and the Solow Residual. With $\phi > 0$, Z is no longer aggregate total factor productivity, because adopters and non-adopters pay different prices for the managerial input. With L^ℓ and X as in Appendix B.4.2, and using $W/P_m^1 = \left(\Omega_0/\Omega_1\right)^{1/\alpha}$,

$$\int m_f \, df = \left(A\eta/\Omega_0\right)^{\frac{1}{1-\eta}} \Omega_0 \frac{\alpha}{W} \int \frac{W}{P_m^{d_f}} \hat{Z}_f^{\frac{\eta}{1-\eta}} \, df = \left(A\eta/\Omega_0\right)^{\frac{1}{1-\eta}} \Omega_0 \frac{\alpha}{W} \left(1 - \chi + \chi\left(\frac{\Omega_0}{\Omega_1}\right)^{\frac{1}{\alpha}}\right) \int \hat{Z}_f^{\frac{\eta}{1-\eta}} \, df.$$

Proceeding as in Appendix B.4.2,

$$Y = \frac{Z}{\left(1 - \chi + \chi\left(\Omega_0/\Omega_1\right)^{1/\alpha}\right)^{\alpha}} \left(L^\ell\right)^{1-\alpha-\gamma} \left(\int m_f \, df\right)^{\alpha} X^{\gamma}.$$

The measured residual therefore equals Z only if $\Omega_0 = \Omega_1$, as at $\phi = 0$, or if no firm adopts.

Decomposition. In the decomposition (38), Z_f is replaced by $\left(\Omega_0/\Omega_1\right)^{d_f} Z_f$, and $\bar{\tau}$ is as defined here. Since Z_f^0 is evaluated without non-rival management, component (2) and the composition of adopters, $\mathcal{R}$, are unaffected. The cost advantage enters Λ, since $\left(\Omega_0/\Omega_1\right)^{d_f}$ is correlated with $\tau(z_f)$ through adoption, and the gain from adoption becomes

$$\mathcal{G} = \left(\frac{\Omega_0}{\Omega_1}\right)^{\frac{\eta}{1-\eta}} \frac{\mathbb{E}_1\left[Z_f^{\frac{\eta}{1-\eta}}\right]}{\mathbb{E}_1\left[\left(Z_f^0\right)^{\frac{\eta}{1-\eta}}\right]} - 1.$$

The gain from adoption therefore combines the productivity effect of non-rival management with the cost advantage.

B.4.4. Aggregation with Product-Level Distortions

This subsection extends the aggregation in Appendix B.4.2 to the product-level wedges of Appendix B.4.1. As in Section 6.1, I set $\phi = 0$, so that $\Omega_0 = \Omega_1 = \Omega$, and use $P = 1$.

Price Equation. With $P = 1$, $Y = \int \int_0^{n_f} p_{i,f} y_{i,f} \, di \, df$. Multiplying and dividing the revenue of each product by its wedge, and using the definitions of $c_{i,f}$ and $\bar{\tau}_f$,

$$Y = \int \int_0^{n_f} \frac{1}{\tau_{i,f}} \tau_{i,f} p_{i,f} y_{i,f} \, di \, df = A \left(A\eta/\Omega \right)^{\frac{\eta}{1-\eta}} \int \frac{1}{\bar{\tau}_f} \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df.$$

Since $A = Y^{1-\eta}$, this gives the aggregate price equation

$$\left(\frac{\Omega}{\eta} \right)^{\frac{\eta}{1-\eta}} = \int \frac{1}{\bar{\tau}_f} \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df,$$

which is the equation with firm-level wedges, with $\bar{\tau}_f$ in place of $\tau(z_f)$. Define $\bar{\tau}$ and Z accordingly,

$$\frac{1}{\bar{\tau}} = \int \frac{1}{\bar{\tau}_f} \frac{\tilde{Z}_f^{\frac{\eta}{1-\eta}}}{\int \tilde{Z}_{f'}^{\frac{\eta}{1-\eta}} \, df'} \, df, \qquad Z^{\frac{\eta}{1-\eta}} = \bar{\tau}^{-\frac{1}{1-\eta}} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df,$$

so that $\Omega = \eta \bar{\tau} Z$.

Factor Share. Variable costs are a share η of the revenue the firm receives, so aggregate variable costs are

$$\eta \int \int_0^{n_f} \tau_{i,f} p_{i,f} y_{i,f} \, di \, df = \eta A \left(A\eta/\Omega \right)^{\frac{\eta}{1-\eta}} \int \tilde{Z}_f^{\frac{\eta}{1-\eta}} \, df,$$

and dividing by Y gives a factor share of $\eta\bar{\tau}$ and an aggregate markup of $\frac{1}{\eta\bar{\tau}}$. The remaining steps in Appendix B.4.2 use only the price equation and the factor share, so $Y \propto Z^{\frac{1}{1-\gamma}} \bar{\tau}^{\frac{\gamma}{1-\gamma}}$ holds with $\bar{\tau}$ and Z as defined here.

Between- and Within-Firm Misallocation. Substituting $\tilde{Z}_f = \bar{\tau}_f^{1/\eta} \mathcal{U}_f \, Z_f$ from Appendix B.4.1 into the definition of Z,

$$Z = \left(\int \left(\frac{\bar{\tau}_f}{\bar{\tau}} \right)^{\frac{1}{1-\eta}} \mathcal{U}_f^{\frac{\eta}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} \, df \right)^{\frac{1-\eta}{\eta}}.$$

This is the expression in Section 6.1 with two changes. First, $\bar{\tau}_f$ replaces the firm-level wedge $\tau(z_f)$, so $\bar{\tau}_f/\bar{\tau}$ captures misallocation between firms as before. Second, $\mathcal{U}_f \leq 1$ captures misallocation within firms. With firm-level wedges, $\bar{\tau}_f = \tau(z_f)$ and $\mathcal{U}_f = 1$, and the expression reduces to the one in Section 6.1. As with firm-level wedges, the wedges also affect Z_f through the firm's choices of n_f and d_f.

B.4.5. Proof of Proposition 6

Maintain the assumptions of the simplified model, $\delta = 0$, $\sigma_c = 0$, and $f_e = 0$, together with $\phi = 0$, so that $\Omega_0 = \Omega_1 = \Omega$, and $P = 1$. Wedges are $\tau_f = \tau_0 z_f^{-\xi}$ with $\xi < \eta$, $z_f \sim \text{Pareto}(\zeta)$ on $[z_{\min}, \infty)$ with distribution function F_z, and τ_0 adjusts to hold $\bar{\tau} = 1$. The proposition evaluates the derivatives at $\xi = 0$. Let $g = \frac{\rho\alpha\eta}{1-\eta}$ and, for $d \in \{0, 1\}$,

$$\upsilon_d \equiv \frac{1}{\eta} \frac{1 + d \cdot g}{\omega - d \cdot g},$$

so that $\frac{\partial \log Z_f^d}{\partial \xi} = -\upsilon_d \left(\log z_f - \mathbb{E}^S[\log z_f] \right)$ and the partial derivative in the proposition is $\frac{\partial \log \left(Z_f^1 / Z_f^0 \right)}{\partial \xi} = -\left(\upsilon_1 - \upsilon_0 \right) \left(\log z_f - \mathbb{E}^S[\log z_f] \right)$, with $\upsilon_1 - \upsilon_0 = \frac{1}{\eta} \frac{1+\omega}{\omega} \frac{g}{\omega - g}$, where the term in $\mathbb{E}^S[\log z_f]$ is the adjustment of τ_0 in equation (B.7) below. Note that $\eta\omega\upsilon_0 = 1$ and $\eta\omega\upsilon_1 = \frac{\omega(1+g)}{\omega-g} = 1 + \frac{g(1+\omega)}{\omega-g}$. Let Cov^S denote the sales-weighted covariance.

Step 1: Sales-Weighted Representation. With a unit mass of firms, Appendix B.4.2 gives $\tau_f S_f = A \left(A\eta/\Omega \right)^{\frac{\eta}{1-\eta}} \tau_f^{\frac{1}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}}$, so S_f is proportional to $\left(\tau_f Z_f \right)^{\frac{\eta}{1-\eta}}$, with a factor common to all firms. The definition of $\bar{\tau}$ in Section 6.1 therefore reads

$$\frac{1}{\bar{\tau}} = \frac{\mathbb{E}\left[S_f \right]}{\mathbb{E}\left[\tau_f S_f \right]}, \qquad \text{so} \qquad \bar{\tau} = \mathbb{E}^S \left[\tau_f \right].$$

The harmonic mean of the wedges weighted by the revenue firms receive equals their arithmetic mean weighted by sales. Substituting into the definition of Z and holding $\bar{\tau} = 1$, $Z^{\frac{\eta}{1-\eta}} = \mathbb{E}\left[\tau_f^{\frac{1}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} \right] = \mathbb{E}\left[\tau_f^{\frac{\eta}{1-\eta}} Z_f^{\frac{\eta}{1-\eta}} \right]$, where the second equality uses $\mathbb{E}^S\left[\tau_f \right] = 1$. Hence, exactly,

$$\frac{\eta}{1-\eta} \log Z = \log \mathbb{E}\left[\left(\tau_f Z_f \right)^{\frac{\eta}{1-\eta}} \right]. \tag{B.6}$$

Differentiating the logarithm of this expectation weights each firm by its share of the expectation, which is its sales share.

At $\xi = 0$, every firm faces $\tau_f = \tau_0$, and $\bar{\tau} = 1$ gives $\tau_0 = 1$. Differentiate $\mathbb{E}^S\left[\tau_f \right] = 1$ at $\xi = 0$. Since $\tau_f = 1$ for every firm and the sales shares sum to one, the change in the sales shares drops out, so $\mathbb{E}^S\left[\frac{d \log \tau_f}{d\xi} \right] = 0$. Since $\frac{d \log \tau_f}{d\xi} = \frac{d \log \tau_0}{d\xi} - \log z_f$,

$$\frac{d \log \tau_0}{d\xi} = \mathbb{E}^S\left[\log z_f \right] \qquad \text{at } \xi = 0. \tag{B.7}$$

Step 2: Firm-Level Productivity. By Section 6, the firm solves the undistorted problem with $\tau_f^{1/\eta} z_f$ in place of z_f. Equation (19) then gives $n_f^d = C_d \left(\tau_f^{1/\eta} z_f \right)^{\frac{\eta}{1-\eta} \frac{1}{\omega - d \cdot g}}$, where $C_d^{\omega - d \cdot g} = (1-\eta)(1+d\cdot g) \left(\eta/\Omega \right)^{\frac{\eta}{1-\eta}} A^{\frac{1}{1-\eta}} / f_c$, using $1 - \eta(1-\rho\alpha) = (1-\eta)(1+g)$. Since $Z_f = z_f n_f^{\frac{1-\eta}{\eta} + d_f \rho\alpha}$,

$\frac{\eta}{1-\eta}\log Z_f = \frac{\eta}{1-\eta}\log z_f + (1+d_f\cdot g)\log n_f$, and

$$\frac{\eta}{1-\eta}\log Z_f^d = \frac{\eta}{1-\eta}\frac{1+\omega}{\omega - d\cdot g}\log z_f + \frac{1}{1-\eta}\frac{1+d\cdot g}{\omega - d\cdot g}\log \tau_f + (1+d\cdot g)\log C_d.$$

From Appendix B.4.2, $\Omega/\eta = \bar{\tau}Z$ and $A^{\frac{1}{1-\eta}} = Y \propto Z^{\frac{1}{1-\gamma}}\bar{\tau}^{\frac{\gamma}{1-\gamma}}$. With $\bar{\tau}=1$,

$$\log C_d = \frac{1}{\omega - d\cdot g}\left(\frac{1}{1-\gamma} - \frac{\eta}{1-\eta}\right)\log Z + \text{const}_d = \frac{1}{(1-\eta)(\omega - d\cdot g)}\left(\frac{1-\eta}{1-\gamma} - \eta\right)\log Z + \text{const}_d.$$

Substituting $\log\tau_f = \log\tau_0 - \xi\log z_f$ and differentiating,

$$\frac{d\log Z_f^d}{d\xi} = \upsilon_d\left\{\frac{d\log\tau_0}{d\xi} + \left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi} - \log z_f\right\}, \tag{B.8}$$

which is equation (41) at $d=0$, and equation (42) since $\upsilon_1/\upsilon_0 = \frac{\omega(1+g)}{\omega - g}$. Holding τ_0 and Z fixed gives the partial derivatives in the proposition.

Step 3: Adoption Rate. By the proof of Proposition 3, at $\phi = 0$ adoption depends on the firm's effective core productivity, $\tau_f^{1/\eta}z_f$, which is increasing in z_f since $\xi < \eta$. The cutoff satisfies

$$\log\bar{z} + \frac{1}{\eta}\log\tau(\bar{z}) = \text{Const}(\rho) + \frac{1-\eta}{\eta}\log\left(\frac{f_c}{A^{\frac{1}{1-\eta}}}\right) + \log\left(\frac{\Omega}{\eta}\right).$$

Substituting $\tau(\bar{z}) = \tau_0\bar{z}^{-\xi}$ and the aggregates as in Step 2,

$$\left(1 - \frac{\xi}{\eta}\right)\log\bar{z} = -\frac{1}{\eta}\log\tau_0 - \frac{1}{\eta}\left(\frac{1-\eta}{1-\gamma} - \eta\right)\log Z + \text{const}.$$

Differentiating, $(\eta-\xi)\frac{d\log\bar{z}}{d\xi} = \log\bar{z} - \frac{d\log\tau_0}{d\xi} - \left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi}$. For $\bar{z} > z_{\min}$, the adoption rate is $\bar{d} = (z_{\min}/\bar{z})^{\zeta}$, so $\frac{d\log\bar{d}}{d\xi} = -\zeta\frac{d\log\bar{z}}{d\xi}$, which gives equation (40). Comparing with equation (B.8) at $z_f = \bar{z}$ and $d=0$,

$$\frac{d\log\bar{d}}{d\xi} = \omega\frac{\eta\zeta}{\eta - \xi}\frac{d\log Z_f^0(\bar{z})}{d\xi}. \tag{B.9}$$

Step 4: Jump at the Threshold. With $\delta = 0$, the first-order condition for scope gives $f_c n_f^{1+\omega} = (1-\eta)(1+d\cdot g)\,\tau_f S_f^d$, where $\tau_f S_f^d$ is the revenue the firm receives under regime d. Profits are therefore

$$\Pi_f^d = (1-\eta)\tau_f S_f^d - \frac{f_c}{1+\omega}n_f^{1+\omega} = \frac{(1-\eta)(\omega - d\cdot g)}{1+\omega}\tau_f S_f^d.$$

With $f_e = 0, \Pi_f^1 = \Pi_f^0$ at $z_f = \bar{z}$, so $(\omega - g)\tau_f S_f^1 = \omega \tau_f S_f^0$ there. Since $\tau_f S_f^d = A\left(A\eta/\Omega\right)^{\frac{\eta}{1-\eta}} \tau_f^{\frac{1}{1-\eta}} \left(Z_f^d\right)^{\frac{\eta}{1-\eta}}$, and τ_f and the aggregates are the same under both regimes at $\bar{z}$,

$$\Delta = \log Z_f^1(\bar{z}) - \log Z_f^0(\bar{z}) = \frac{1-\eta}{\eta} \log\left(\frac{\omega}{\omega - g}\right).$$

The same condition gives the jump in sales at the threshold, $\bar{S}^1/\bar{S}^0 = \frac{\omega}{\omega - g} = e^{\frac{\eta}{1-\eta}\Delta}$, so that

$$\Delta^S = \frac{1-\eta}{\eta} \frac{\bar{S}^1 - \bar{S}^0}{\mathbb{E}\left[S_f\right]} = \frac{\bar{S}^0}{\mathbb{E}\left[S_f\right]} \frac{1-\eta}{\eta} \left(e^{\frac{\eta}{1-\eta}\Delta} - 1\right),$$

which equals $\frac{\bar{S}^0}{\mathbb{E}[S_f]}\Delta$ to first order in Δ.

Step 5: Aggregate Productivity. Split the expectation in equation (B.6) at the threshold,

$$\mathbb{E}\left[\left(\tau_f Z_f\right)^{\frac{\eta}{1-\eta}}\right] = \int_{z_{\min}}^{\bar{z}} \left(\tau_f Z_f^0\right)^{\frac{\eta}{1-\eta}} dF_z + \int_{\bar{z}}^{\infty} \left(\tau_f Z_f^1\right)^{\frac{\eta}{1-\eta}} dF_z.$$

Differentiate with respect to ξ. Within each regime, the change in $\log\left(\tau_f Z_f\right)$ is weighted by the sales share. At the threshold, the integrand jumps from a value proportional to $\bar{S}^0$ to one proportional to $\bar{S}^1$. Using $\frac{d\bar{d}}{d\xi} = -F_z'(\bar{z})\frac{d\bar{z}}{d\xi}$ since $\bar{d} = 1 - F_z(\bar{z})$,

$$\frac{d\log Z}{d\xi} = \mathbb{E}^S\left[\frac{d\log\tau_f}{d\xi}\right] + \mathbb{E}^S\left[\frac{d\log Z_f}{d\xi}\right] + \Delta^S \frac{d\bar{d}}{d\xi},$$

where $d\log Z_f/d\xi$ is evaluated at the firm's own d_f. The last term is the extensive margin: each firm crossing the cutoff adds Δ^S. At $\xi = 0$, the first term is zero by equation (B.7).

Consider the second term. By equations (B.8) and (B.7),

$$\mathbb{E}^S\left[\frac{d\log Z_f}{d\xi}\right] = \mathbb{E}^S\left[\upsilon_{d_f}\right]\left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi} - \mathrm{Cov}^S\left(\upsilon_{d_f}, \log z_f\right),$$

where $\mathbb{E}^S\left[\upsilon_{d_f}\right] = (1-\bar{d}^S)\upsilon_0 + \bar{d}^S\upsilon_1$. Since $\upsilon_{d_f} = \upsilon_0 + d_f\left(\upsilon_1 - \upsilon_0\right)$ and $\frac{\partial\log\left(Z_f^1/Z_f^0\right)}{\partial\xi}$ differs from $-\left(\upsilon_1 - \upsilon_0\right)\log z_f$ by a constant that does not affect the covariance,

$$-\mathrm{Cov}^S\left(\upsilon_{d_f}, \log z_f\right) = \mathrm{Cov}^S\left(d_f, \frac{\partial\log\left(Z_f^1/Z_f^0\right)}{\partial\xi}\right) = \bar{d}^S\,\mathbb{E}_1^S\left[\frac{\partial\log\left(Z_f^1/Z_f^0\right)}{\partial\xi}\right] = \mathcal{I},$$

where the second equality uses $\mathbb{E}^S\left[\frac{\partial\log\left(Z_f^1/Z_f^0\right)}{\partial\xi}\right] = 0$, since the partial derivative is proportional to $\log z_f - \mathbb{E}^S[\log z_f]$.

Consider the third term. By equation (40) at $\xi = 0$ and equation (B.7),

$$\frac{d\bar{d}}{d\xi} = \bar{d}\,\frac{\zeta}{\eta}\left(\mathbb{E}^S\left[\log z_f\right] - \log \bar{z} + \left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi}\right) = \bar{d}\,\frac{\partial \log \bar{d}}{\partial \xi} + \bar{d}\,\frac{\zeta}{\eta}\left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi}.$$

Multiplying by Δ^S,

$$\Delta^S\,\frac{d\bar{d}}{d\xi} = \mathcal{E} + \bar{d}\,\frac{\zeta}{\eta}\Delta^S\left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\xi}.$$

Collecting the terms in $\frac{d\log Z}{d\xi}$,

$$(1-\mathcal{M})\,\frac{d\log Z}{d\xi} = \mathcal{I} + \mathcal{E},$$

where

$$\mathcal{M} = \left(\frac{1-\eta}{1-\gamma} - \eta\right)\left[(1-\bar{d}^S)\upsilon_0 + \bar{d}^S\upsilon_1 + \bar{d}\,\frac{\zeta}{\eta}\Delta^S\right] = \frac{1-\eta}{\eta}\frac{1}{\omega}\left(\frac{1}{1-\gamma} - \frac{\eta}{1-\eta}\right)\left(1 + \bar{d}^S\,\frac{g(1+\omega)}{\omega - g} + \bar{d}\,\omega\zeta\,\Delta^S\right),$$

the second equality using $\eta\omega\upsilon_0 = 1$ and $\eta\omega\upsilon_1 - 1 = \frac{g(1+\omega)}{\omega-g}$. This is the roundabout multiplier in the proposition. As equation (B.9) shows, the adoption rate moves with the response of the marginal adopter, so $\mathcal{E}$ weights the jump Δ^S by the size of the marginal adopter relative to the sales-weighted average firm. Dividing by $1-\mathcal{M}$ gives equation (43). ■

Decomposing the Multiplier. By equation (B.8) and Step 3, $\frac{\partial \log Z_f^d}{\partial \log Z} = \left(\frac{1-\eta}{1-\gamma} - \eta\right)\upsilon_d$, so $\frac{\partial \log\left(Z_f^1/Z_f^0\right)}{\partial \log Z} = \left(\frac{1-\eta}{1-\gamma} - \eta\right)(\upsilon_1 - \upsilon_0)$, and, at $\xi = 0$, $\frac{\partial \log \bar{d}}{\partial \log Z} = \left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{\zeta}{\eta}$. Since $(1-\bar{d}^S)\upsilon_0 + \bar{d}^S\upsilon_1 = \upsilon_0 + \bar{d}^S(\upsilon_1 - \upsilon_0)$, the expression for $\mathcal{M}$ above is equation (44). At $\rho = 0$, $g = 0$, so $\bar{S}^1 = \bar{S}^0$ and $\Delta^S = 0$. Then $\mathcal{M}|_{\rho=0} = \frac{1}{\eta\omega}\left(\frac{1-\eta}{1-\gamma} - \eta\right)$, and both $\mathcal{I}$ and $\mathcal{E}$ are zero: at $\xi = 0$, the distortion moves Z at first order only through non-rival management. The result is exact and does not approximate the distribution of firms.

B.4.6. Adoption and the Common Wedge

This subsection derives the elasticity of the adoption rate with respect to the common wedge τ_0, holding ξ fixed, in the simplified model with $\phi = 0$. Unlike in Proposition 6, $\bar{\tau}$ is not held fixed. Substituting $\Omega/\eta = \bar{\tau}Z$ and $A^{\frac{1}{1-\eta}} = Y \propto Z^{\frac{1}{1-\gamma}}\bar{\tau}^{\frac{\gamma}{1-\gamma}}$ (Appendix B.4.2) into the cutoff condition in Step 3 of Appendix B.4.5, and keeping the terms in $\bar{\tau}$,

$$\left(1 - \frac{\xi}{\eta}\right)\log \bar{z} = -\frac{1}{\eta}\log \tau_0 - \frac{1}{\eta}\left(\frac{1-\eta}{1-\gamma} - \eta\right)\log Z - \frac{1}{\eta}\left(\frac{\gamma(1-\eta)}{1-\gamma} - \eta\right)\log \bar{\tau} + \text{const.}$$

Differentiating with respect to $\log \tau_0$ and using $\frac{d\log \bar{d}}{d\log \tau_0} = -\zeta\frac{d\log \bar{z}}{d\log \tau_0}$,

$$\frac{d\log \bar{d}}{d\log \tau_0} = \frac{\zeta}{\eta - \xi}\left\{1 + \left(\frac{1-\eta}{1-\gamma} - \eta\right)\frac{d\log Z}{d\log \tau_0} + \left(\frac{\gamma(1-\eta)}{1-\gamma} - \eta\right)\frac{d\log \bar{\tau}}{d\log \tau_0}\right\}. \tag{B.10}$$

The interpretation is the same as for equation (40). The first term is the direct effect: a higher τ_0 raises the revenue the marginal adopter receives and lowers the cutoff. The second is the feedback through aggregate productivity. Relative to equation (40), $\bar{\tau}$ is no longer held fixed, so the third term captures the effect of the factor-share wedge on adoption, which operates through the wage and market size.

To first order, $\log \bar{\tau} \approx \mathbb{E}[\log \tau_f] = \log \tau_0 - \xi\, \mathbb{E}[\log z_f]$ (Step 1 of Appendix B.4.5), so $\frac{d \log \bar{\tau}}{d \log \tau_0} = 1$. The direct and factor-share terms then combine, since $1 + \frac{\gamma(1-\eta)}{1-\gamma} - \eta = \frac{1-\eta}{1-\gamma}$, and

$$\frac{d \log \bar{d}}{d \log \tau_0} \approx \frac{\zeta}{\eta - \xi} \left\{ \frac{1-\eta}{1-\gamma} + \left(\frac{1-\eta}{1-\gamma} - \eta \right) \frac{d \log Z}{d \log \tau_0} \right\}.$$

Holding Z fixed, a higher τ_0 therefore raises the adoption rate.

B.5. Quantitative Implementation and Additional Results

Figure B1 plots the model analogue of the US data in Figure 2 under the baseline calibration.

Figure B1: Firm Scale and Scope: Model

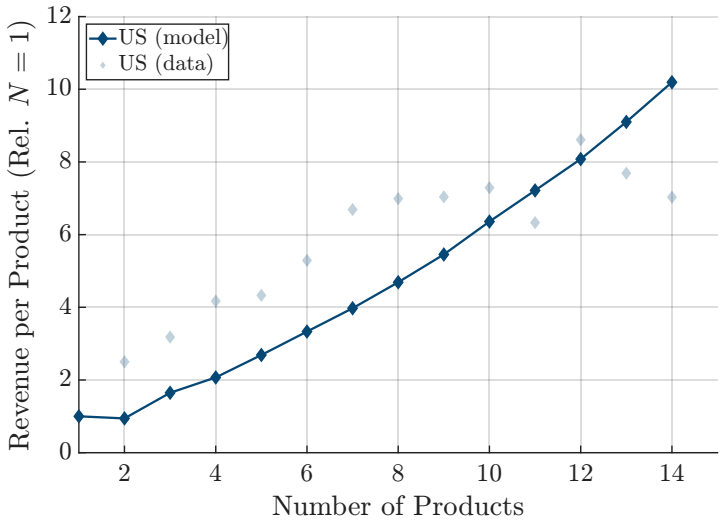


Table B1 reports the parameters, targeted moments, and firm-size distribution of the calibration in which ρ is fixed at zero.

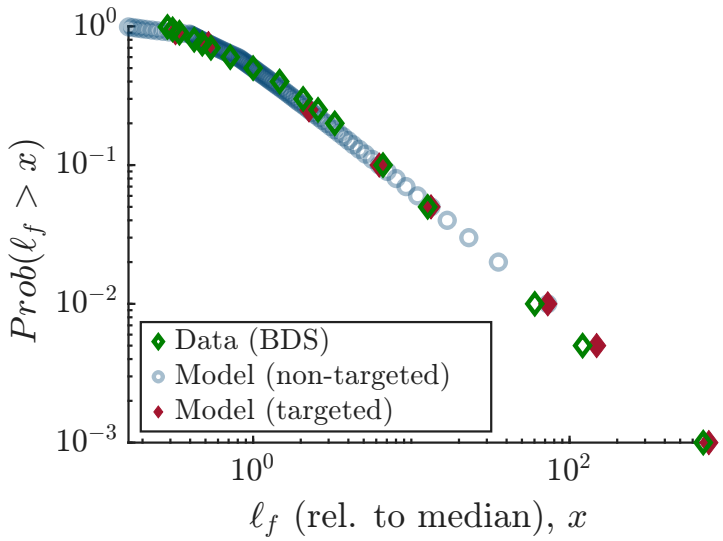
Table B2 reports the parameters and targeted moments of the calibration to India in which the scope and core-competency parameters are recalibrated.

Table B3 reports the analogous calibration to India starting from the $\rho = 0$ calibration in Table B1.

Table B1: Calibration with $\rho = 0$

Parameters			*Targeted moments*		
Param.	Description	Value	Moment	Data	Model
Calibrated			*Scope distribution*		
ω	Scope-cost curvature	1.048	n_{p99}	14.00	14.09
μ_c	Mean of $\log f_{c,f}$	-2.417	$n_{p99.5}$	21.00	19.52
σ_c	S.d. of $\log f_{c,f}$	0.950	$n_{p99.9}$	41.00	41.43
ζ	Pareto shape of z_f	2.643	$\mathbb{E}(n_f)$	2.16	2.32
δ	Core competency	0.295	$\mathbb{V}\text{ar}(\log n_f)$	0.67	0.75
			Firm-size distribution		
			Employment, p10–p99.9	Panel (b)	
			Regression coefficients		
			β_{core}	-0.59	-0.59

(a) Parameters and moments



(b) Firm-size distribution

Notes: US data. Calibration with ρ fixed at zero, using the same strategy and data as the baseline (Tables 5 and 6). Assigned parameters are as in the baseline. With $\rho = 0$, non-rival management brings no gain, so f_e and β_{IV} drop out of the calibration. Panel (b) plots the complementary CDF of employment relative to the median.

Table B2: Calibration to India

Parameters			*Targeted moments*		
Param.	Description	Value	Moment	Data	Model
Calibrated			*Scope distribution*		
μ_c	Mean of $\log f_{c,f}$	-3.567	n_{p99}	9.00	9.15
σ_c	S.d. of $\log f_{c,f}$	1.053	$n_{p99.5}$	11.00	11.31
ξ	Wedge elasticity	0.112	$n_{p99.9}$	18.00	18.36
τ_0	Wedge level	0.846	$\mathbb{E}(n_f)$	2.01	1.62
δ	Core competency	0.504	$\mathbb{V}\text{ar}(\log n_f)$	0.61	0.45
			Firm-size distribution		
			ℓ_{p99}/ℓ_{p50}	9.50	11.63
			$\ell_{p99.5}/\ell_{p50}$	21.00	18.26
			$\ell_{p99.9}/\ell_{p50}$	86.00	55.73
			Regression coefficients		
			β_{IV}	0.01	0.01
			β_{core}	-1.00	-1.00

Notes: Parameters not listed are held at their US values (Table 5). Firm size is employment from the 2005 Economic Census. Scope moments are derived from Prowess.

Table B3: Calibration to India with $\rho = 0$

Parameters			*Targeted moments*		
Param.	Description	Value	Moment	Data	Model
Calibrated			*Scope distribution*		
μ_c	Mean of $\log f_{c,f}$	-3.304	n_{p99}	9.00	9.30
σ_c	S.d. of $\log f_{c,f}$	1.197	$n_{p99.5}$	11.00	11.15
ξ	Wedge elasticity	0.216	$n_{p99.9}$	18.00	16.92
τ_0	Wedge level (sets $\bar{\tau}$)	0.980	$\mathbb{E}(n_f)$	2.01	2.08
δ	Core competency	0.500	$\mathbb{V}\text{ar}(\log n_f)$	0.61	0.64
			Firm-size distribution		
			ℓ_{p99}/ℓ_{p50}	9.50	13.36
			$\ell_{p99.5}/\ell_{p50}$	21.00	20.71
			$\ell_{p99.9}/\ell_{p50}$	86.00	56.04
			Regression coefficients		
			β_{core}	-1.00	-1.00

Notes: Parameters not listed are held at their values in the US calibration with $\rho = 0$ (Table B1). Firm size is employment from the 2005 Economic Census. Scope moments are derived from Prowess.